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THE UNIVERSITY OF ALBERTA

A REPORT ON THE ALGEBRA 2 PAPERS OF 1940, 1941 AND 1942

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR

THE DEGREE OF MASTER OF EDUCATION

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CHAPTER I

PURPOSE OF THE INVESTIGATION

Two thousand five hundred and sixty-nine students wrote the Algebra 2 examination in June 1940. Eight hundred and seven of these students secured less than one quarter of the total possible mark. Two thousand two hunred and thirteen students made less than half the total possible mark. Twenty-nine students secured seventy-five per cent or more of the total possible score.

The results in 1941 were very much the same as in 1940. In 1942 the results were much better but a different type of examination was given.

The purpose of this investigation is to analyse the errors made by the students and so discover, if possible, the cause of these low scores. The examination papers will be criticized and suggestions offered.

CHAPTER II

PROCEDURE FOLLOWED IN CARRYING OUT THE STUDY

The Faculty of Education made arrangements with the Department of Education to obtain the sample papers. For each year there were approximately three hundred sample papers.

The method of procedure was simple. One problem was considered at a time. For example, Problem 1 on the 1940 paper was taken. Every answer paper was studied and all the various solutions recorded. The total number making each type of error was found and listed as a per cent. Where only two or three papers showed the same type of errors they were classified as miscellaneous errors.

The same procedure was carried out for each of the three papers. Comments were made on the results obtained. The results were given as percentages under the following headings:

(a) Correct Solution

(b) Did not Attempt the Problem

(c) Common Errors

(d) Miscellaneous Errors

There were 297 papers in the sample lot for the 1940 examination, 309 for the 1941 examination, and 322 for the 1942 examination. The percentages given are based on these totals but should be reasonably true for all the answer papers. What the probable error is, in using these samples instead of all the answer papers, is shown in the following cases.

Assume problems not attempted as incorrect.

Sample lot 300 papers.

Case 1.   60% correct   40% incorrect

Standard Deviation from correct score is

$$\sqrt{\frac{60 \times 40}{300}} = 2\sqrt{2}\%$$

Since the sample is large, the distribution under simple sampling conditions is normal and the range of the score is from $60 - 3(2\sqrt{2})$ up to $60 - 3(2\sqrt{2})$

That is, the probable range is from 52 to 68 per cent correct.

The maximum range would occur when 50% were correct. In this case the s. d. would be $\sqrt{\frac{2500}{300}} = \sqrt{\frac{25}{3}}$ %, which is not much greater than that shown in the above case.

Case 2.   90% correct   300 sample papers

Standard Deviation is $\sqrt{\frac{90 \times 10}{300}} = 3$

The range is from $90 - 3\sqrt{3}$ to $90 - 3\sqrt{3}$

That is, the range is from 85 to 95.

These illustrations show that for all practical purposes the percentages obtained from the samples are reliable.

CHAPTER III

TABLE I

Results of 1940 Examination

Frequency Table

0 - 9	56
10 - 19	208
20 - 29	380
30 - 39	489
40 - 49	459
50 - 59	391
60 - 69	258
70 - 79	155
80 - 89	103
90 - 99	41
100 - 109	16
110 - 119	9
120 - 127	4
128 - 134	0
	2569 answer papers

Question 1

If y varies as x^2 what is the effect

(a) on y of dividing x by 9;

(b) on x of dividing y by 9?

Solution		%
(a)	Correct	20
	Did not try it	8
	y is divided by 3	17
	Indefinite terms but correct idea	14
	y is divided by 9	15
	y is multiplied by 9	6
	y is multiplied by 81	5
	Miscellaneous errors	15
(b)	Correct	25
	Did not try it	11
	x is divided by 81	17
	Indefinite terms but correct idea	11
	x is divided by 9	10
	x is multiplied by 9	7
	Miscellaneous errors	19
Both parts correct		15
Both parts correct and over 50 total marks		92

On the average, only one student in seven could correctly solve both parts of this problem. More surprising still, about one in ten did not even attempt it.

The problem could be solved easily in two ways. They could use the method of generalization or they could take special cases.

Many of those who solved the problem, simply wrote the answer so they may have obtained it by guessing. Nearly everyone realized that the answer would involve 9, $\sqrt{9}$, or 9^2, but the incorrect guesses were almost as numerous as the correct ones.

Question 2

Sketch the graph of $y = 5x - 4 - x^2$ and find the areas between the x-axis and the portion of the curve above the x-axis.

Solution	%
Graph	
Correct	65
Did not try it	14
Miscellaneous errors	21
Integration	
Correct	42
Did not try it	14
Mechanical errors	22
Miscellaneous errors	22
Both parts correct	22
Both parts correct and over 50 total marks	85

Many of the students made mechanical errors and so got entirely unreasonable answers. They failed to use the graph to test the correctness of their results.

For practical purposes, the best method to use would be the approximate result as found from the graph. It can be found quite accurately and it is the simplest method. The problem does not state the method of solution required and yet no marks were allowed for this solution. If more emphasis were placed on approximate answers and less on mechanical solutions by formulae, we probabley would get much better results.

The sketch of the graph was fairly well done. Most of the difficulty arose from failure to note the significance of the minus sign in front of x^2. That is, the student showed a line which continued to rise.

Question 3

The weights of two spheres are in the ratio 20:9 and the denisities of the materials of which they are made are in the ratio 5:18. What is the ratio of the radii?

Solution	%
Correct	7
Did not try it	45
Mechanical errors	6
Correct formula but did not complete work	8
Used $\frac{r_1^2}{r_2^2}$ (ratio of areas)	6
Used $\frac{r_1}{r_2}$ (ratio of lengths)	16
Miscellaneous errors	12
Correct solution and over 50 total marks	95

This question is very fair. There is a problem practically identical to this one in the text. The solution does not require the memorization of a formula as it depends solely on the knowledge that volumes of similar figures are to one another as the cube of corresponding sides.

There are five problems in the section on Ratio that compare volume of similar objects. There are also several of these problems in the Revision exercises. This should be sufficient to make clear to the student how the principle actually works.

Nearly half of the students did not attempt the problem. This probably indicates that the students are not sure of themselves in problems involving ratio. The large

number comparing volumes of similar objects as being directly proportional to their radii are probably just following the line of least resistance. The numbers are given, so without thinking how they should be used, the students put them down in the simplest manner possible. The students get an answer of 8 to 1, which seems correct to them because it happens to be a ratio of simple whole numbers.

Of those who made mechanical errors several failed to find $\sqrt[3]{8}$ correctly. Two students said that $\sqrt[3]{8}$ was equal to $2\sqrt{2}$.

Question 4

The enrolment in a certain school is as shown in the table.

	Grade XI	Grade XII
Boys	16	14
Girls	18	9

In how many ways may a committee of six be chosen if there are to be on the committee two boys and one girl from Grade XII, and one boy and two girls from Grade XI?

Solution	%
Correct	17
Did not try it	16
Took Combinations in XII Combinations in XI	14
Took number of combinations in each group	
91 + 9 + 16 + 153 = 269	14
Permutation	7
Mechanical errors	17
Miscellaneous errors	15
Correct and over 50 total marks	70

The students generally consider this section of the text relatively easy. This is shown here by the large number attempting the problem. The poor results are probably due to the fact that many students depend too much on the answer to show them how to work the problem. The student has a general idea of the method. He tries one method of solution and checks with the answer in the text. The correct answer generally indicates any error he may have made. The student learns to manipulate numbers so that his

answer will check with that given in the book but he does not really comprehend how the problem should be solved.

There were a very large number of mechanical errors in the work on this problem. Practically all of these errors were in multiplication.

If a problem such as: "What is the value of $10\ C_4$?" is given, practically all of the students will solve it correctly. That is, they have learned how to solve a type problem but they fail to understand the principle; so when a problem is varied slightly, they do not know what to do.

Question 5

Given $y = x^2$

(a) Find $\frac{dy}{dx}$ for this function.

(b) When $x = 1.5$, evaluate $\frac{dy}{dx}$.

(c) When $x = 1.5$ and $\Delta x = 0.2$,

(1) find Δy;

(2) find $\frac{dy}{dx} \Delta x$;

(3) find $\Delta y - \frac{dy}{dx} \Delta x$.

(d) What is the average gradient of the curve between $x = 1.5$ and $x = 1.7$?

(e) What is the gradient of the curve at (1.5, 2.25)?

Solution

		%
(a)	Correct	95
	Did not try it	2
	$\frac{dy}{dx} = \frac{1}{2x}$	1
	Miscellaneous	2
(b)	Correct	96
	Did not try it	2
	Miscellaneous	2
(c)	(1) Correct	42
	Did not try it	10
	$\frac{dy}{dx} \simeq \frac{y}{x} \therefore y = 2 \times 1.5x \times .2 = .6$	23
	$\frac{\Delta y}{\Delta x} = 2x = 2 \times 1.5 = 3$	10

Graph	1
Miscellaneous errors	14
(2) Correct	51
Did not try it	21
Wrong answer; no work	24
Miscellaneous errors	4
(3) Correct	29
Did not try it	26
Wrong answer; no work	23
Mechanical errors	3
Miscellaneous errors	19
(d) Correct	44
Did not try it	25
Multiplied .2 by 1.5	2
Found difference in value of y when x = 1.5 and x = 1.7 but did not calculate slope	17
Mechanical error	1
Graph	3
Miscellaneous errors	8
(e) Correct	30
Did not try it	43
$\frac{1.5}{.75}$	12
$\frac{2.25}{1.5}$	2
Miscellaneous errors	13
All parts correct	17
All correct and over 50 total marks	72

There was no difficulty evaluating $\frac{dy}{dx}$. Practically all students can solve simple differentiation if there are no difficulties in indices present.

The chief error in part "C" was in taking $\frac{dy}{dx} \simeq \frac{\Delta x}{\Delta y}$ and so finding the value of Δy. This is a logical error since they have just been asked to find $\frac{dy}{dx}$. To avoid this error the question might be worded: "Find Δy from first principles."

The relatively large number of errors in "C (2)" as compared with the errors in "b" is due largely to difficulty with "c (1)".

About one half of those who got "d" correct, solved it by finding the value of y for the two given values of x, and so finding the average slope. The others found $\frac{\Delta y}{\Delta x}$ using first principles.

Those who solved the problem graphically received no credit for their effort; yet some received half credit for an answer of 32, when the correct answer was 3.2. I believe that an answer which is approximately correct, as found from a graph, is worth more than an unreasonable answer obtained from the correct formula.

96% of the students solved "c" correctly. That is they could find the value of $\frac{dy}{dx}$ but only 30% knew what it was when they had found it.

Question 6

Given x : y = 2 : 5 and y : z = 4 : 3 and z : w = 4 : 5 express in its simplest form x : y : z : w.

Solution	%
Correct	16
Did not try it	12
Not complete but correct as far as it goes 8 : 20 : 15 ___	28
Mechanical errors	16
Just started it	10
Miscellaneous errors	18
Correct and over 50 total marks	85

Many students stopped working when they saw that they were not getting ratios expressed in small whole numbers. The example in the text has the ratio given as 10 : 15 : 21 : 14. The students apparently thought they had made an error when they did not get small numbers in their ratio and so did not try to finish the problem.

16% of the students made mechanical errors. This was much the same as in problem 4. Both problems involved approximately the same amount of mechanical work. This means that one may expect to find one error in six where the work requires several multiplications to be made.

Question 7

If $y = \frac{4}{x^3} + \frac{6}{\sqrt{x}}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, expressing the results without negative indices.

Solution		%
(a)	Correct	45
	Did not try it	5
	$\frac{dy}{dx}\frac{4}{x^3} = \frac{4}{3x^2}$	5
	$\frac{dy}{dx} 4x^{-3} = -12x^{-2}$	3
	Error in changing $-3x^{-3/2}$ to form $\frac{-3}{\sqrt{x^3}}$	18
	Error in sign only	8
	Mechanical errors	7
	Miscellaneous errors	9
(b)	Correct	40
	Did not try it	5
	Could not solve second part since first part was wrong	21
	Error in sign	15
	Mechanical errors	10
	Miscellaneous errors	9
Both parts correct		33
Both parts correct and over 50 total marks		61

The chief difficulty in this problem is the correct use of indices. Fractional and negative indices are not understood very well.

21% of the students could not solve the second part of the problem due to trailing error. Problems of this type should be avoided as much as possible.

This type of problem does not require much thought. The students learn the rule and then apply it mechanically. The fact that only 5% did not try it indicates that the students like this type of problem.

Question 8

A piece of wire 20 inches long is bent so as to form a rectangle. One side of the rectangle is x inches and the area of the rectangle is A square inches.

(a) Express A as a function of x.

(b) Graph A against x.

(c) Find the range of values of x for which A is an increasing function of x.

(d) Find the range for which A is decreasing.

(e) Find the value of x for which A is a maximum.

Solution		%
(a)	Correct	75
	Did not try it	10
	$A \propto xy$	5
	$A = 20x - x^2$	4
	$A \propto x$	2
	$A \propto \frac{1}{x}$	1
	Miscellaneous	3
(b)	Correct	56
	Did not try it	12
	No idea of critical points	12
	Straight line graph	6
	Poor graph, partial marks	4
	Miscellaneous	10
(c)	Correct	49
	Did not try it	18

	A increases from 1 to 5 (Trouble with $x = 0$)	12
	A increases from $x = -\infty$ to $+5$	6
	Error in use of $>$ and $<$ to express limits	5
	Miscellaneous	10
(d)	Correct	52
	Did not try it	18
	Error in use of $>$ and $<$ to express limits	4
	A decreases as x goes from 5 to $+\infty$	9
	A decreases from $x = 5$ to $x = 9$ (difficulty with $x = 10$ and $a = 0$)	7
	A decreases only when A is negative	5
	Miscellaneous	5
(e)	Correct, no method shown	36
	Correct by graph	9
	Correct by use of first and second differential	28
	Did not try it	16
	Miscellaneous	11
All parts correct		35
All parts correct and over 50 total marks		90

The results on this problem were fairly good. Trailing error was again quite evident in this case. Those who had "a" part wrong generally had all the rest of the question wrong.

The students did not do very well on the graph. They don't know what to do with x = 0. Many of them broke off the graph at x = 1 and x = 9. Those who gave the correct solution for "e" but did not state how they got it probably used their graph.

Question 9

(a) Given $y = z^4$ and $z = x^3 - 5x^2 + 4$, find $\frac{dy}{dx}$ in terms of x

(b) Use the method of (a) to find $\frac{d}{dx}(3x + 1)^2$.

(c) Use some other method to find $\frac{d}{dx}(3x + 1)^2$.

Solution		%
(a)	Correct	13
	Did not try it	35
	Multiplied it but made mechanical error then differentiated	27
	Multiplied it and got correct result	1
	Mechanical error only	8
	Miscellaneous errors	16
(b)	Correct	13
	Did not try it	47
	Multiplied and made mechanical error	14
	Multiplied and got correct result	23
	Miscellaneous errors	3
(c)	Correct by multiplication	21
	Correct by first principles	16
	Did not try it	46
	Multiplied and made mechanical error	5
	Integrated	3
	Miscellaneous	9
All parts correct		9
All parts correct and over 50 total marks		96

Apparently very few of the students remembered the first rule of differentiation in respect to three variables: "If y is a function of x and x is a function of t, the rate at which y changes is given by $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$"

28% of the students tried to find the value of $(x^3 - 5x^2 + 4)^4$ by multiplication. When you consider the marks given for the problem this method is obviously not a good one. Of all those trying this method less than 1% managed to get the correct result. Probably only the weaker students used this method but the percentage of mechanical errors was certainly very high.

The problem is relatively easy in comparison with those given in the text but all of this section on differentiation proves to be very difficult for the students.

Question 10

$y = 2x^3 - 15x^2 + 36x.$

(a) Find the co-ordinates of each of the points on the curve at which the tangent is horizontal.

(b) Find one point at which the tangent has a negative slope.

(c) Find the co-ordinates of a minimum point.

(d) How do you know that the point in (c) is a minimum point?

(e) Find the co-ordinates of a point which is on the curve and also on the x-axis.

(f) Draw a freehand sketch of the curve.

Solution

		%
(a)	Correct	34
	Did not try it	50
	By graph, partial marks	4
	Mechanical error	4
	Integrated	2
	$\frac{d^2y}{dx^2} = 0$	2
	Miscellaneous	4
(b)	Correct	25
	Did not try it	62
	Gave a minimum	3
	Wrong answer no work	7
	Just said "where $\frac{dy}{dx}$ is negative"	3

(c)	Correct	29
	Did not try it	59
	Mechanical error	3
	Gave both maximum and minimum	3
	$\frac{d^2y}{dx^2} = 0$	2
	Miscellaneous errors	4
(d)	Correct	23
	Did not try it	58
	$\frac{dy}{dx} = 0$	8
	When $\frac{d^2y}{dx^2}$ is positive	8
	Miscellaneous	3
(e)	Correct	31
	Did not try it	62
	Miscellaneous errors	7
(f)	Correct	32
	Did not try it	53
	Very poor graph	15
All parts correct		18
All parts correct and over 50 total marks		82

The large number who did not try this will be made up of two groups. One will consist of those who are rushed for time and so are looking for easier problems. The others are ones who realize that this type of problem is too difficult for them.

Some students answered part "d" as follows: "The point is a minimum because it is lower than any point on either side of it." Of course, the answer expected, was in terms of the first and second derivative. This answer shows that the student understands the question and probably deserves credit.

15% of the graphs drawn received no credit and deserved none. They were freehand sketches in the full sense of the word. There were a fairly large number of straight line graphs.

Question 11

The tangent of the curve $y = \frac{100}{x}$ at the point C (5, 20) meets the x-axis at A and the y-axis at B; P is any point on the curve.

(a) Sketch the graph of the curve.

(b) What is the gradient of the tangent at P $(a, \frac{100}{a})$?

(c) What is the gradient of the tangent at C?

(d) What is the gradient of the straight line through C and A?

(e) What is the equation of the tangent to the curve at C?

(f) What are the co-ordinates of A?

(g) What is the area of $\triangle$AOB?

Solution		%
(a)	Correct	22
	Did not try it	56
	Just the part in the first quadrant	20
	Miscellaneous	2
(b)	Correct	12
	Did not try it	77
	Error in sign	4
	Value from the graph	3
	Did not use $x = a$	3
	Miscellaneous	1
(c)	Correct	17
	Did not try it	71

	Error in sign	4
	Approximate value from graph	3
	Inverse slope	3
	Miscellaneous errors	2
(d)	Correct	7
	Did not try it	81
	Approximate value from graph	4
	Error in sign	2
	Inverse slope	2
	Miscellaneous errors	4
(e)	Correct	5
	Did not try it	87
	$y = ax + b$	3
	$y = \frac{-100}{x^2}$	2
(f)	Correct	5
	Did not try it	86
	Approximate value from graph	5
	Miscellaneous errors	4
(g)	Correct	6
	Did not try it	88
	Approximate value from graph	4
	Miscellaneous errors	2
All parts	correct	3
All parts	correct and over 50 total marks	100

A large number of students did not try this question. Since a much greater number tried the following problems, this is an indication that the students consider this type of problem to be quite difficult.

Students, as a rule, tend to avoid problems involving graphs. I believe that the topic of graphs has been handled quite badly all through the grades.

Probably one of the main reasons why graphs have not been used and understood better by students is illustrated here. In practically all parts of the question some students gave an approximate result obtained from the graph. In all cases no marks were given for their effort. I believe that the students who can draw graphs and get an area or slope approximately correct should be given credit for their work. They show that they have understood what they have done. Part marks are given for problems with ridiculous answers where the correct method has been used and a mechanical error has been made. In my opinion this policy is bad.

Question 12

(a) Solve $\frac{dy}{dx} = x^5 + 2x^2 - 3$.

(b) What is the solution of (a) for which $y = 5$ when $x = 2$?

Solution		%
(a)	Correct	54
	Did not try it	22
	Forgot the " + C"	13
	Differentiated	7
	Miscellaneous errors	4
(b)	Correct	24
	Did not try it	39
	Couldn't solve the problem because they forgot the " + C"	13
	Mechanical error	12
	Error in sign only	7
	Miscellaneous errors	5
Both parts correct and over 50 total marks		73

The problem is a standard one and should have been easy enough for the majority of the students.

The chief error in "a" was in forgetting the constant. This trailing error affected "b" thus resulting in a double penalty.

The number of mechanical errors was high. This shows up in all problems where much mechanical work is required.

Question 13

(a) What is the average of the numbers in the series 8, 11, 14, 17, 20, ... 341, 344?

(b) If t_n is the n^{th} term of the series in (a) above, express t_n as a linear function of n, i.e. $t_n = (?) + (?)n$.

(c) Using a (first term), l (last term) and n, find an expression for S_n the sum of n terms of an arithmetic series.

(d) Use (c) to find the sum of the series in (a)

Solution

		%
(a)	Correct	66
	Did not try it	9
	Mechanical error	5
	Found number of terms	4
	Miscellaneous	16
(b)	Correct	15
	Did not try it	56
	Gave general formula $t_n = a + (n - 1)d$	9
	$t_n = 8 + 3n$	8
	$t_n = a + (t_n - 1)d$	6
	$t_n = 8 + 3(n - 1)$	3
	$t_n = 8 + (t - 1)n$	1
	Miscellaneous errors	2
(c)	Correct	76
	Did not try it	19
	Formula for sum of a Geometric series	2

	Formula for last two terms of a series	1
	Miscellaneous errors	2
(d)	Correct	26
	Did not try it	31
	No of terms wrong $\left(\frac{l - a}{d} = 112\right)$	4
	No of terms wrong not getting 112 as answer	11
	Mechanical errors	8
	Found number of terms, not sum	5
	Incomplete answer e.g. 176n	6
	Miscellaneous	9
All parts correct		6
All parts correct and over 50 total marks		99

It was surprising to find that only two thirds of the students could find the average of these numbers. This may have been due to the number of marks given for the question. The correct solution would seem too easy for the number of marks given.

Part "b" is well worded but the fact that in all previous problems of this type the students had been asked to find the n^{th} term, made them think this was a different type of problem.

Part "c" is merely a test of memory work. Most of the students memorize formulae even if they do not understand them. This is shown in part "d". The students knew the formula but could not apply it to solve the problem.

Question 14

(a) How many terms are there in the expansion of $(1 + x)^{2n-1}$, if n is a positive integer?

(b) Use two terms of a binomial expansion to find an approximate value of $\sqrt[3]{1729}$.

(c) By what per cent does this cube root exceed $\sqrt[3]{1728}$?

(d) Write the fifth term of the expansion of $(1 - x)^{1/2}$.

Solution

		%
(a)	Correct	25
	Did not try it	42
	Infinite number	8
	There are an indefinite number of terms	6
	Took a definite value of n and then gave the number of terms in this case but did not see how to get the correct result from this.	5
	$2n - 1$	3
	Miscellaneous	11
(b)	Correct	0
	Did not try it	72
	$(1000 + 729)^{1/3}$, accepted as partially correct	4
	Mechanical error with above method	5
	$(1 + 1728)^{1/3} = 1 + 1/3 \times 1728 = 577$	6
	$(1730 - 1)^{1/3}$	5
	$1000 + 1/3 \times 729 = 1243$	2

	Miscellaneous	6

Some of the miscellaneous answers are

$1/3 \times 1700 \times 29$

$(1700 + 29)^{1/3}$

1729^3 and then multiplied to get the result

$\sqrt[3]{1728} + \sqrt[3]{1} = 12 + \sqrt[3]{1}$

(c)	Correct using 12.43 as $\sqrt[3]{1729}$	3
	Did not try it	89
	Mechanical error	1
	$(1730 - 2)^{1/3}$	1
	$(1700 + 28)^{1/3}$	2
	Miscellaneous	4
(d)	Correct	9
	Did not try it	53
	Mechanical error	12
	Took sixth term	10
	Wrong formula	4
	$\frac{1}{2}C_4x^4$	3
	Took fourth term	2
	Miscellaneous	7

Of those who had "a" part correct, very few showed how they got their result. 5% of the students could not get the general solution after solving the special case.

Part "c" proved too difficult for the students. The examiners expected the students to know the value of $\sqrt[3]{1728}$ but apparently the students did not know this. Probably the

students did know the answer in relation to cubic measure, but not having the cue, failed to think of it.

Another surprise was the relatively large number giving an answer of 577 or 1243. Apparently these students saw nothing amiss with these results. The students have not been trained to check their results against an estimated answer to see if their final answer is reasonable.

In "d" part, more students gave the sixth term than the fifth. Probably they neglected to count "1" as a term and only thought of terms involving x.

Question 15

Deduce from the appropriate series the formula for each of the following. Do not evaluate.

(a) The amount of \$400 in 3 years with interest at 5 per cent per annum, compounded half-yearly.

(b) The cash value of an annuity of \$2000 running 20 years, first payment due six years from now, money worth 5%.

(c) The annuity, first payment due one year hence, which at 3% will accumulate to \$5000 ten years from now.

Solution		%
(a)	Correct	6
	Did not try it	33
	Miscellaneous answers given partial credit	10
	$400\,(1.05^6 - 1)$	7
	$\frac{400}{1.05}(1 - \frac{1}{1.05^5})$	5
	$400\,\frac{(1.025)^6 - 1}{1.025 - 1}$	4
	Miscellaneous errors mostly with no idea of correct formula	35
(b)	Correct	25
	Did not try it	35
	$\dfrac{\frac{2000}{1.05}\left[1 - \frac{1}{1.05^{14}}\right]}{1 - \frac{1}{1.05}}$	6
	Mechanical error only	5
	Miscellaneous	29

(c)	Correct	14
	Did not try it	46
	Miscellaneous	40

Of the miscellaneous answers three or more students gave the following as their result.

$$\frac{5000}{(1.03)^{10} - 1}$$

$$\frac{5000}{1.03}\ \left(\frac{1 - \frac{1}{1.05^{10}}}{1 - \frac{1}{1.05}}\right)$$

$$\frac{\left[5000 - \frac{1}{1.03}\ (5000)\right] X\ 1.03}{1 - \frac{1}{1.03^{10}}}$$

$$5000\ \left(\frac{1.03^{10} - 1}{1.03 - 1}\right)$$

$$5000 = x\left(\frac{1 - 1.03^{-10}}{1.03 - 1}\right)$$

$$5000 = \frac{x}{1.03^2}\left(\frac{1 - \frac{1}{1.03^{11}}}{1 - \frac{1}{1.03}}\right)$$

$$x = \frac{5000}{1.03\left(\frac{1.03^{10} - 1}{1.03 - 1}\right)}$$

$$\frac{x}{1.03}\left[\frac{1 - \frac{1}{1.03^{10}}}{1 - \frac{1}{1.03}}\right]$$

All parts correct	1
All parts correct and over 50 total marks	100

With the exception of part "c" the results are very bad. In the case of "c" an identical example is worked in the text with the exception of the amount. Then, too, the

1938 paper has this problem on it and of course the students work out old examination papers. I believe that the majority who solved "c", did so by memorizing the form and did not really know what it implied.

The solutions listed for part "c" illustrate quite well what the final result is. The students try to memorize a complicated form and end up thoroughly muddled.

Question 16

Given four points representing pairs of observations: (1, 88), (2, 41), (4, 16) and (5, 19). A relation of the form $y = a + b(\frac{1}{x})$ is suggested.

(a) Plot y against $\frac{1}{x}$.

(b) Find a relation approximately expressing y in terms of x.

Solution

		%
(a)	Correct	10
	Did not try it	65
	Plotted y against x and took values of y corresponding to $y = \frac{1}{x}$ for various values of x.	22
	Miscellaneous errors	3
(b)	Correct	1
	Did not try it	59
	Mechanical error	10
	Took two points of y plotted against x and got correct equation of this line	19

$$a + \frac{b}{1} = 88$$

$$a + \frac{b}{2} = 41$$

$$\therefore \frac{b}{2} = 47$$

$$b = 94$$

$$a = -6$$

$$\therefore y = \frac{94}{x} - 6$$ 5

$\dfrac{\frac{1}{x} - \frac{1}{x_1}}{\frac{1}{x_1} - \frac{1}{x_2}} = \dfrac{y - y_1}{y_1 - y_2}$	2
Miscellaneous	4
Both parts correct	1
Both parts correct and over 50 total marks	100

As stated before it does not require the results of a departmental examination to show that students do not understand graphs. If a graph is simple they can draw it but they do not understand it. A previous study of mine, "Graphs in General Mathematics" illustrated this fact quite well.

Question 17

Using the same axes, and the same scales on both the x and the y axes throughout, draw the graphs of

(a) $y = f(x)$ where $f(x) = (x - 1)^2(x + 2)$;

(b) $y = f'(x) = \frac{d}{dx}\left\{(x - 1)^2(x + 2)\right\}$

(c) $y = f''(x) = \frac{d}{dx}\left[\frac{d}{dx}\left\{(x - 1)^2(x + 2)\right\}\right]$

(d) What information about $f(x)$ is given by the graph of $f'(x)$?

(e) What information about $f'(x)$ is given by the graph of $f''(x)$?

Solution		%
(a)	Correct	29
	Did not try it	34
	No idea of critical points	25
	Correct but did not consider negative values of x	12
(b)	Correct	27
	Did not try it	38
	Attempt made but no idea of correct graph	23
	Correct but did not consider negative values of x	12
(c)	Correct	33
	Did not try it	51
	Attempt made but no idea of correct graph	15

	Straight line graph but wrong slope	1
(d)	Correct	1
	Did not try it	69
	It tells whether it is a maximum or minimum	8
	It is a curve	4
	Graph of f'(x) is the tangent of graph of f(x)	2
	The values of y when x = 0	2
	Miscellaneous	14
(e)	Correct	1
	Did not try it	76
	It tells us if there is a maximum or a minimum	2
	Worked out actual values of y = f"(x)	10
	It is a straight line	5
	Miscellaneous	6

All parts correct (Less than 1%)

A large number of students did not try this question. Some of these probably did not have time to try it.

Of those who drew the graphs very few knew what information could be obtained from them.

CHAPTER IV

Results of 1941 Examination

Question 1

(a) $\frac{d}{dx}x^3 =$

(b) $\int x^3 dx =$

(c) t varies as r. If r increases, t ----

(d) The maximum value of $5 - x^2$ is ----

(e) $y = x^{-2/3}$. If $x = 8$, $y =$

(f) If $f(x) = \log x$, then $f(1) =$

(g) In the graph of $y = \frac{1}{x}$, the sign of the slope is ----

(h) As x increases without limit, $\frac{2x - 3}{5x}$ approaches the limit ----

(i) If $\frac{dy}{dx} = 0$ at a given point on a curve, then the tangent to the curve at that point is ----

(j) If $\frac{dy}{dx}$ is negative, then y is a ... function.

Solution		%
(a)	Correct	92
	$2x^2$	4
	$3x^4$	2
	Integrated	1
	Miscellaneous errors	1
(b)	Correct	24
	Did not try it	4

	$\frac{1}{4}x^4$ (forgot to add the constant)	54
	$3x^4$	3
	$\frac{3}{4}x^4$	3
	$\frac{x^2}{3}$	2
	Miscellaneous errors	10
(c)	Correct	87
	Did not try it	0
	Decreases	10
	Miscellaneous errors	3
(d)	Correct	48
	Did not try it	10
	Zero	5
	$5x - 1/3\ x^3$	2
	When $x = 1$ Maximum value is 4	6
	Maximum value is $-2\frac{1}{2}$	10
	Maximum value is 2	5
	Maximum value is -5	3
	Maximum value is 5	3
	Miscellaneous errors	8
(e)	Correct	62
	Did not try it	5
	$y = +4$	11
	$y = -5\ 1/3$	7
	$y = 4^{-1}$ (no marks given)	4
	$y = \frac{1}{\sqrt{512}}$	2
	Miscellaneous	9

(f)	Correct	89
	Did not try it	7
	Miscellaneous	4
(g)	Correct	63
	Did not try it	6
	Positive	23
	$-\frac{1}{x^2}$ (no marks given)	8
(h)	Correct	22
	Did not try it	10
	The limit is zero	20
	The limit is infinity	13
	The limit is one	12
	The limit is -1/5	11
	The limit is 3	7
	The limit is -3	3
	Miscellaneous	2
(i)	Correct	49
	Did not try it	26
	Tangent is equal to zero	20
	Tangent is a straight line	3
	Miscellaneous	2
(j)	Correct	39
	Did not try it	18
	y is negative	22
	y is positive	9

y is a maximum	8
Miscellaneous errors	4
All parts correct	1
All parts correct and over 50 total marks	100

Since answers only were required, we can only infer how the incorrect answers were obtained. In some cases the incorrect method used is quite obvious.

The marking was not done any too well in this question. In part "e" the answer 4^{-1} was marked incorrect but since the problem is stated using a negative index surely the answer is correct using the same form. In part "g" a fairly large number of students gave answers of "$-\frac{1}{x^2}$" or "-" and received no credit. In the first case they gave more than was required, in the second case they used a symbol instead of a word. In both cases I see no reason for marking them as incorrect.

Part "b" was badly done. Most of the errors were due to forgetting to add the constant. This was a common error on problem 12 in 1940.

Part "d" proved to be quite difficult. Students experience difficulty whenever zero is encountered. Of those who stated that the maximum value is zero probably some really meant that the maximum would occur when $x = 0$. 10% gave the maximum value as -2 1/2. It appears to have been obtained by making an equation, $-2x = 5$, and solving it. Those who gave the answer "4" took $x = \pm 1$. They took as small a value of x as possible and yet avoided fractions and zero.

On part "d" I made a special check. Of those whose

marks were above the median 80% got this part correct. Of those whose marks were below the median 22% got this part correct. Briefly, this problem rates high as a test of the students' ability as shown by the results of the examination.

It is quite likely that many of the correct answers in part "g" were obtained by chance. When one considers this, the results are quite poor. This is another instance of weakness in graphical work.

Part "h" could have been solved easily if the students had taken special values of x. Pupils should be encouraged to take specific cases and so determine the correct solution.

Parts "i" and "j" indicate that students can get answers much better than they can explain results. They have learned to manipulate figures but often don't understand what the figures mean.

Question 2

A bullet, fired vertically upwards, rises s feet in t seconds, where $s = 500\,t - 16\,t^2$.

(a) When $t = 4$ and $\Delta t = 0.1$, evaluate $\frac{\Delta s}{\Delta t}$

(b) When $t = 4$, evaluate $\frac{ds}{dt}$.

(c) In terms of the motion of the bullet, interpret $\frac{\Delta s}{\Delta t}$ and $\frac{s}{t}$, attaching the correct unit in each instance.

Solution		%
(a)	Correct	28
	Did not try it	24
	Mechanical error	18
	$\frac{\Delta s}{\Delta t} = \frac{ds}{dt} = 372$	15
	$\frac{\Delta s}{\Delta t} = \frac{500 - 32\,t}{\Delta t} = 3720$	7
	$2000 - 256 = 1744$	5
	Miscellaneous errors	3
(b)	Correct	81
	Did not try it	16
	Mechanical error	3
(c)	Correct	12
	Did not try it	46
	Vague idea, partial credit	14

$\frac{\Delta s}{\Delta t}$ is average slope between 4 and 4.1 and $\frac{ds}{dt}$ is the slope at 4	8
$\frac{\Delta s}{\Delta t}$ correct $\frac{ds}{dt}$ wrong	7
Used graph to illustrat $\frac{\Delta s}{\Delta t}$	3
Miscellaneous errors	10
All parts correct	7
All parts correct and over 50 total marks	95

This is a good practical problem. It is worded well and if the students understand differentiation it should cause no difficulty. The results show that the students can use a formula to get a result but that they cannot interpret this result.

Question 3

If a, 3, 4, b are in continued proportion, find $\frac{a}{b}$.

Solution	%
Correct	21
Did not try it	22
$\frac{a}{b} = \frac{4}{3}$	20
$\frac{a}{b} = \frac{3}{4}$	13
$ab = 12 \therefore \frac{a}{b} = \frac{12}{1}$	8
$\frac{a}{b} = \frac{4/3}{3/4}$	5
$\frac{a}{b} = \frac{12}{12b}$	3
Miscellaneous	8
Correct and over 50 total marks	75

The chief problem here was in understanding what was given. That is, the students did not know how to write the problem as continued proportion. If the problem had been given as $\frac{a}{3} = \frac{3}{4} = \frac{4}{b}$, probably most of the students could have solved it.

Question 4

Expressing results without negative or fractional exponents, find $\frac{dy}{dx}$ when

(a) $y = \frac{x^3}{6} + 4\sqrt{x} - \frac{1}{2x^2}$

(b) $y = 2\sqrt{x^3} - \frac{3}{\sqrt[3]{x}}$

Solution		%
(a)	Correct	30
	Did not try it	39
	$\frac{x^2}{2} + \frac{2}{\sqrt{x}} + \frac{4}{x^3}$	12
	$\frac{x^2}{2} + \frac{2}{\sqrt{x}} + \frac{1}{\sqrt[3]{x}}$	8
	$\frac{x^2}{2} + \frac{2}{\sqrt{x}} - \frac{4}{x^3}$	4
	$\frac{x^2}{2} + \frac{1}{\sqrt{x}} + \frac{1}{4x^3}$	2
	Miscellaneous	5
(b)	Correct	34
	Did not try it	32
	$3\sqrt{x} - \frac{2}{\sqrt[3]{x^5}}$	10
	$\frac{-3}{\sqrt{x}} - \frac{1}{\sqrt[3]{x^4}}$	10
	$\frac{2}{3\sqrt[3]{x^2}} + \frac{1}{\sqrt[3]{x^4}}$	5
	Miscellaneous	9
Both parts correct		9
Both parts correct and over 50 total marks		95

Students always seem to experience difficulty with negative and fractional indices. It was surprising to find more students attempt the second part than the first.

Practically all of the errors in the first part were made in dealing with the last term. The students apparently cannot express $\frac{-1}{2x^2}$ in the form $\frac{-x^{-2}}{2}$.

The results are not as good as they were in 1940 on a similar problem.

Question 5

A varies as $\frac{W}{v^2}$ where A sq. ft. is the area of the wing surface of an aeroplane, W tons is its total weight (aeroplane and load), and v miles per hour is its speed.

(a) Find the ratio of the wing areas of two aeroplanes, each having a total weight of 3 tons, the first having a speed of 100 miles per hr. and the second a speed of 120 miles per hr.

(b) One aeroplane has a wing area of 810 sq. ft. and a speed of 150 miles per hr. A second aeroplane has a wing area of 750 sq. ft. and a speed of 180 miles per hr. Find the ratio of the total weight of these aeroplanes.

(c) If the total weight of an aeroplane is increased in the ratio of 16:9, how must the speed be altered, and in what ratio, so that the wing area need not be changed?

Solution		%
(a)	Correct	44
	Did not try it	31
	Mechanical errors	14
	Not simplified	5
	$\frac{36}{25} = \frac{6^2}{5^2} = \frac{6}{5}$	1
	Miscellaneous	5
(b)	Correct	30
	Did not try it	30
	Mechanical errors	21

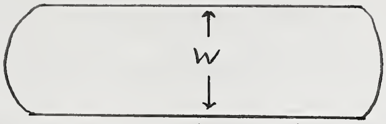
	Not simplified	10
	Partially correct	7
	Miscellaneous	2
(c)	Correct	29
	Did not try it	46
	Inverse ratio	9
	Not simplified	4
	Mechanical error	5
	Miscellaneous errors	7
All parts correct		15
All parts correct and over 50 total marks		95

This problem involved quite a lot of reading in order to find out what was required. The problem is up-to-date and should arouse pupils' interest. The good student finds little difficulty with this type of problem. Its solution seems to depend more on mental ability than on learning a set method.

The marking could have been better on this problem. There were several instances of correct results with no marks given. Then there were many papers with correct results but the answer was not simplified. For example, $\frac{27}{36}$ was given instead of $\frac{3}{4}$. Many of these were given no credit, some were given 1 mark and some were given 2 marks. Some correct results were given 2 instead of 3 marks.

Question 6

A design for a flower bed is shown in the following figure, which consists of a rectangle and two semi-circles. Its perimeter is 100 ft.



(a) If the area of the figure is A sq. ft. and its width W ft., express A as a function of W.

(b) If W = 6, compute the area of the bed to the nearest square foot.

(c) Show the range of values of W thus .....<W<.....

Solution

		%
(a)	Correct	12
	Did not try it	65
	$50W - W^2$	10
	Not simplified $\left[\pi\left(\frac{W}{2}\right)^2 + W\left(100 - \frac{\pi W}{2}\right)\right]$	9
	Miscellaneous	4
(b)	Correct	10
	Did not try it	67
	$50W - W^2 = 300 - 36 = 264$	10
	Mechanical error	2
	Trailing error	11
(c)	Correct	3
	Did not try it	79
	$0<W<100$	6
	Miscellaneous	12

Some of these miscellaneous answers were:

$12 < W < 12$; $0 < W < 66\ 2/3$; $0 < W < 25$;

$50 < W < +1$; $6 < W < 44$.

Those few having all parts correct had high scores.

This problem proved to be very difficult for the students. The problem does not depend upon memorizing some intricate formula. Certainly most of the students know how to find the circumference and area of a circle when given the diameter.

The students lack power of visualization. They have not learned to make quick sketches to aid them in problem solving.

Trailing error is introduced again in this problem. If the first part is not correct the students cannot solve part "b". It is very difficult to avoid all cases of trailing error but I believe it should be avoided as much as possible.

Part "c" should be easy enough for any grade twelve student but only 3% got it correct. 79% of the students did not try the problem. Possibly this section of the text has proven so difficult that the students have become confused. They look for a difficulty which does not exist.

It would be interesting to find what per cent of the grade nine students could solve this problem. If a fair number of them could solve it we could assume that this topic had not been treated well in grade twelve.

Question 7

(a) Under what conditions is $1 + nx$ a close approximation for $(1 + x)^n$?

(b) Use (a) to find approximate values of

(1) $\frac{1}{\sqrt{0.98}}$

(2) $\sqrt[3]{27.81}$

Solution		%
(a)	Correct	14
	Did not try it	47
	When n is small	16
	When x is small	14
	When $n = 1$	5
	When nx is small	2
	When n is small compared with x	2
(b)	Correct	32
	Did not try it	49
	$1 - 1/2 \times .98 = .51$	6
	$\frac{1}{\sqrt{.98}} = 1.98^{1/2} = (1 + .98)^{1/2} = 1.49$	2
	$\frac{1}{\sqrt{.98}} = (1 - .02)^{-1} = 1.02$	2
	Mechanical errors	4
	Miscellaneous	5
(c)	Correct	14
	Did not try it	46
	Error in use of decimal	10
	$\sqrt[3]{27.81} = (1 + 26.81)^{1/3} = 9.93$	9
	$\sqrt[3]{27.81} = (27 + .81)^{1/3} = 27.27$	4

$\sqrt[3]{27.81} = 3\sqrt[3]{1.81} = 3(1 + .81)^{1/3} =$	
$3(1 + .27)^{1/3} = 3.81$	8
$\sqrt[3]{27.81} = 1 + 1/3(27.81) = 10.27$	2
Miscellaneous errors	7
All parts correct	3
All parts correct and over 50 total marks	100

This problem proved to be unusually difficult. In part "a" the rule is given but the students clearly do not know the conditions under which it applies.

In part "b" the rule can be used almost directly to get the correct result, so a fair number of students succeeded in getting it.

In part "c" where the problem had to be changed, so that it would take the same form as the example much difficulty was experienced. Over 20% of the students gave ridiculous answers for part "c" and apparently did not notice anything wrong with them.

By merely glancing at the problem the student should realize that the answer should be slightly over 3. If he fails to get this result, when he calculates it, he should certainly check his work for errors.

Question 8

(a) If $\frac{dy}{dx} = 2x^3 + \frac{2}{x^3}$, and $y = 7$ when $x = 2$, express y as a function of x.

(b) Solve: $\frac{ds}{dt} = \sqrt{t^3} - \frac{1}{\sqrt[3]{t}}$

Solution		%
(a)	Correct	34
	Did not try it	30
	$y = \frac{1}{2}x^4 - \frac{1}{2x^4} + 7$	8
	$y = \frac{1}{2}x^4 - x^{-2}$	10
	Forgot constant	7
	Miscellaneous	11
(b)	Correct	17
	Did not try it	39
	Error in sign	18
	Forgot the constant	9
	$(2/5\ \sqrt{t^5} - 3/2\ \sqrt[3]{t^2} + c)$	7
	Miscellaneous	10
Both parts correct		16
Both parts correct and over 50 total marks		83

The problem if more difficult than the one given the previous year. Students have a great deal of difficulty with negative indices. This probably accounts for the 20% lower number of correct answers. It would have been better, perhaps, to give one rather simple problem in integration as well as the more difficult one, to find out if they understood the principle.

Question 9

If $y = 1 + 3x - 5x^2$, and x changes from 1 to 1.02,

(a) Find the exact value of $\triangle y$.

(b) Use differentiation to find an approximate value for $\triangle y$.

(c) Calculate, correct to one decimal place, the percentage error in the approximation obtained in (b).

Solution		%
(a)	Correct	16
	Did not try it	50
	Error in sign	6
	Did not consider $-5h^2$, just worked out first two terms	7
	Mechanical error	7
	Miscellaneous errors	14
(b)	Correct	30
	Did not try it	48
	Mechanical error	9
	Miscellaneous errors	13
(c)	Correct	14
	Did not try it	58
	Part "a" or "b" wrong so could not solve "c"	28
All parts correct		8
All parts correct and over 50 total marks		100

The results on this problem are much lower than on a similar problem in 1940. Apparently teachers are not stressing the method of determining Δy.

Mechanical errors are quite frequent. It is quite evident that accuracy and checks are not given major emphasis.

Question 10

Four boys and five girls are eligible to hold office in a student organization.

(a) In how many ways can an executive consisting of president, vice-president and secretary be chosen?

(b) In how many ways can this executive be chosen if the president must be a boy and the vice-president a girl?

(c) In how many ways can a social committee of five members be chosen if there must be at least one boy on the committee?

Solution		%
(a)	Correct	18
	Did not try it	18
	$9C_3 = 84$	20
	$4C_3 + 4C_2 \times 5C_1 + 4C_1 \times 5C_2 + 5C_3 = 84$	12
	$4C_3 \times 5C_3 = 40$	8
	Mechanical errors	6
	Miscellaneous errors	18
(b)	Correct	26
	Did not try it	24
	$4C_2 \times 5C_2 = 60$	8
	$4C_1 + 5C_1 + 7C_1 = 16$	6
	$9P_1 \times 4P_1 \times 5P_1 = 180$	6
	$4C_1 \times 5C_1 \times 3C_1 \times 4C_1 = 240$	4
	$4C_1 \times 5C_2 \times 9C_1 = 360$	3
	$4P_2 \times 5P_2 = 240$	3

	Miscellaneous	20
(c)	Correct	16
	Did not try it	34
	$5P_1 \times 4P_1 = 20$	6
	$8C_4 = 70$	5
	$9P_5 = 126$	5
	$4C_1 \times 5P_4 = 480$	3
	$5C_4 = 5$	3
	Miscellaneous	28
All parts correct		3
All parts correct and over 50 total marks		100

The students generally like these problems. There is a considerable jump in the number who tried this problem over the number who tried the three preceding questions.

In part "a" more students tried to solve the problem as a combination than as a permutation. They failed to realize that the same group of three could vary as to president, vice-president, and secretary.

One answer for "b" was 100,800 and one for "c" was 237,120. That is, they do not realize when the result is ridiculous.

The problem was quite fair. The first two parts should not have caused difficulty if the students really understood this section of the text. The last part was more difficult but serves well to test the better students.

Question 11

(a) Evaluate $\int_{-3}^{1}(x^2 + 1)dx$.

(b) Interpret geometrically your result in (a), using a graph sketch.

Solution		%
(a)	Correct	58
	Did not try it	18
	Mechanical errors	13
	Subtracted value when x = 1 from value when x = -3 to get -10 2/3	5
	Errors in integration	6
(b)	Correct	30
	Did not try it	47
	Drew graph $y = 1/3\ x^3 + x + c$	10
	No graph but correct explanation	5
	Correct graph but no explanation	3
	Miscellaneous errors	5
Both parts correct		28
Both parts correct and over 50 total marks		67

Part "a" can be done without much mental effort. Pupils memorize the method and then apply it without much thought as to what is implied. The relatively high percentage correct indicates this quite well. The percentage of mechanical errors is fairly high but not above normal where this amount of work is required.

Though only 30% got part "b" correct this is much higher than usual for a problem of this nature. The students recognize their weakness as is shown by the large percentage of those who did not try it.

Question 12

In the expansion of $(2x - \frac{y}{2})^7$

(a) How many terms contain both x and y?

(b) What is the exponent of y in the term containing x^3?

(c) Write down the 4th term in the form $K x^m y^n$.

Solution		%
(a)	Correct	44
	Did not try it	42
	7 terms	6
	5 terms	6
	Miscellaneous	2
(b)	Correct	46
	Did not try it	39
	The exponent of y is 3	5
	The exponent of y is 5	4
	Gave coefficient of term containing x^3	4
	Miscellaneous	2
(c)	Correct	8
	Did not try it	56
	$35\ x^4\ y^3$	6
	$35\ x^4\ (\frac{-y}{2})^3$	5
	$7\ C_3\ x^4\ y^3$	4
	Included K, m and n	8
	Mechanical error	7
	Miscellaneous	6

All parts correct	6
All parts correct and over 50 total marks	95

This is a standard type problem. It is a fair test of the students' knowledge of the binomial theorem. It was surprising to find so many students not attempting this question. Those who did try, made many errors. The results indicate the type of error made.

Question 13

$y = 6x - x^2 - 5$

(a) Express this function in its factored form.

(b) Sketch its graph.

(c) Determine the gradients of the tangents to the curve when it crosses the x-axis.

(d) Calculate the area of the triangle formed by the tangents mentioned in (c) and the x-axis.

Solution		%
(a)	Correct	54
	Did not try it	35
	$y = (x - 5)(x - 1)$	8
	Miscellaneous	3
(b)	Correct	46
	Did not try it	46
	Graph of: $y = x^2 - 6x + 5$	5
	Other incorrect graphs	3
(c)	Correct	30
	Did not try it	57
	1 and 5	6
	From graph, no marks given	2
	Miscellaneous	5
(d)	Correct	8
	Did not try it	74
	Approximate area from graph	3
	Miscellaneous errors	15

All parts correct	7
All parts correct and over 50 total marks	75

Part "a" should be easy enough for anyone who has taken first year ~~of~~ algebra. Some of those, who did not try it, may ~~not~~ have thought it was not worth trying, since so few marks were given and their time may have been nearly up.

In part "c", 6% of the students just gave the points where the curve crossed the axis. Probably the students were rushed and did not read the question carefully enough to see what was required.

The students who tried part "d" had a great deal of difficulty in finding the height of the triangle. Some of them tried to get the equation of each line and so solve the point of intersection. Many different types of errors were made.

Question 14

A water pitcher is 6 inches high and holds one pint. Another pitcher of the same shape is 9 inches high; find its capacity.

Solution	%
Correct	26
Did not try it	21
$\frac{6}{9} = \frac{1}{x}$, $x = 1\frac{1}{2}$ pts.	38
$\frac{\pi r^2 6}{\pi r^2 9} = \frac{1}{x}$, $x = 1\frac{1}{2}$ pts.	10
$\frac{V_2}{V_1} = \frac{(l_2)^3}{(l_1)^3} = 3$	3
$\frac{9^2}{6^2} = \frac{x}{1}$, $x = 2\frac{1}{4}$	1
Miscellaneous	1
Correct and over 50 total marks	60

This problem is about as easy as it could be and still test the students' ability to compare the volumes of similar objects. It is a good problem and should have been handled better by the students.

Pictograms are very often interpreted wrongly in much the same manner as the students handled this question. In this way many people, by making visual comparisons, get a wrong sense of values.

Question 15

(a) The 8th and 13th terms of an A. P. are 25 and 40 respectively. Find the 10th term.

(b) The sum of the first n terms of a series is $8 - \frac{1}{2}n-3-$

Write down the first three terms.

Solution		%
(a)	Correct	66
	Did not try it	24
	$d = 3 \therefore$ 10th term $= 30$	4
	Mechanical error	6
(b)	Correct	5
	Did not try it	77
	Took $n = 3$ $S_n = 7$	2
	Miscellaneous	16
Both parts correct		4
Both parts correct and over 50 total marks		100

Part "b" proved to be too difficult for most of the students. The students fail to realize that they can substitute any value they wish for n and so find the three terms. Several students let $\frac{n}{2}\left\{2a - (n - 1)d\right\} = 8 - \frac{1}{2}n-3-$ and then tried to solve this equation. They failed to note that they had three unknowns and only one equation.

Question 16

The following table shows a few of the results obtained in an experiment:

x	2.5	3.2	3.8	4.5	5.0
y	0.5	4.0	11.3	14.2	19.5

(a) On graph paper plot y against x^2.

(b) Assuming that there is a linear relationship between y and x^2, state which pair of results is most in error.

(c) Excluding the point indicated in (b), draw the best fitting straight line among the remaining points.

(d) From this line, work out an equation in the form $y = ax^2 + b$.

(e) When x = 4, what is the value of y?

(f) When x = 4, what is the value of $\frac{dy}{dx}$?

Solution		%
(a)	Correct	48
	Did not try it	20
	Correct idea but not accurate	12
	Plotted y against x	20
(b)	Correct	46
	Did not try it	32
	(3.2, 4)	11
	(5, 19.5)	5
	(4.5, 14.2)	4
	Gave three points in error	2
(c)	Correct	43

	Did not try it	29
	Fair but not accurate	6
	y plotted against x so line wrong	15
	y plotted against x but given credit	4
	Very inaccurate	3
(d)	Correct	21
	Did not try it	42
	Mechanical errors	19
	y plotted against x so equation wrong	10
	Used x not x^2	5
	Miscellaneous	3
(e)	Correct	20
	Did not try it	57
	Wrong equation	14
	Mechanical errors	7
	Miscellaneous errors	2
(f)	Correct	19
	Did not try it	65
	Wrong equation	10
	Mechanical error	4
	Miscellaneous errors	2
All parts correct		7
All parts correct and over 50 total marks		100

Most graphs, which the students are required to draw, have y plotted against x. That is probably the reason why so many tried to work this question in this manner.

In part "d" all work reasonably accurate was considered correct. The ones listed as mechanical errors gave results far from correct.

Since "d" part was wrong in many cases, parts "e" and "f" could not be solved correctly. This is another place where trailing error occurs. Students do not clearly understand what the equation $y = mx + b$ represents.

The marking on this problem was certainly not good. In part "a" several students were given credit on the graph part of the paper and also on their foolscap sheet.

On part "b" the point in error is (3.8, 11.3) if one considers y plotted against x. It is (14.44, 11.3) if one considers y plotted against x^2. The markers were not consistent in marking these two answers. In many cases either one was given credit; in other cases either one was marked incorrect.

On part "c" the majority of the students who had plotted y against x were given no credit for the best fitting straight line drawn through these points. Some of them however were given credit for this result.

Question 17

$y = 4x + 2 + \frac{1}{x}$

(a) For the following ranges, state whether $\frac{dy}{dx}$ is positive or negative:

(1) $-3 < x < -\frac{1}{2}$

(2) $-\frac{1}{2} < x < -\frac{1}{4}$

(3) $\frac{1}{4} < x < \frac{1}{2}$

(4) $\frac{1}{2} < x < 3$

(b) Using the results obtained in (a), find all the values of x at which y is maximum or minimum.

(c) Find all the maximum and minimum values of y.

(d) Sketch the graph of y against x.

Solution		%
(a)	Correct	34
	Did not try it	15
	3 parts correct	5
	2 parts correct	41
	1 part correct	3
	None correct all tried	2
(b)	Correct	22
	Did not try it	68
	Miscellaneous errors	10
(c)	Correct	14
	Did not try it	72
	Partly correct	3
	Miscellaneous errors	11

(d)	Correct	5
	Did not try it	66
	Incorrect graph	29
All parts correct		4
All parts correct and over 50 total marks		92

Apparently there was much guessing done on the first part of this question. Most of those who were wrong, stated that the first two parts would be negative and the last two parts would be positive. This answer was favored, probably, because x has negative values in the first two cases and positive values in the last two cases.

Part of the trouble with those who showed their work was in finding $\frac{dy}{dx}$ of $\frac{1}{x}$. This involves the use of the negative index which always seems to confuse students. Trailing error was very much in evidence in this problem.

The graph was very badly done. Since this question is very near the end of the paper; it is quite likely that many of the students did not have time to finish all of the paper; so decided it would take too much time to draw the graph.

Question 18

(a) Find the cash value of $500.00 due in 10 years if money is worth 3%. Give a complete logarithmic solution.

(b) Equal sums of money invested annually for 7 years amount to $3,206.00 one year after the last investment. If interest is calculated at 5% compounded annually, find the annual investment. Give a complete logarithmic solution.

Solution		%
(a)	Correct	5
	Did not try it	56
	Mechanical error	3
	Miscellaneous errors	36
(b)	Correct	1
	Did not try it	73
	Incorrect solution	26

Only one student had all parts correct.

This section of the text always proves to be exceedingly difficult for the students. Less than one per cent of the students got the correct result in part "b".

The answers for part "a" varied from $8 to $22,377.60 and in part "b" from $17.88 to over $3000. Many students put down fantastic answers.

Question 19

(a) Name the curve whose equation is $y = \pm\sqrt{9 - x^2}$.

(b) Evaluate, to four significant figures, $2\int_0^3 \pi(9 - x^2)dx$.

(c) What is the geometrical significance of the result obtained in (b)?

Solution		%
(a)	Correct	22
	Did not try it	59
	Parabola	10
	Hyperbola	5
	Ellipse	4
(b)	Correct	14
	Did not try it	48
	36π	14
	18π	3
	Mechanical error	6
	Miscellaneous errors	15
(c)	Correct	12
	Did not try it	78
	Volume of a circle from 0 to 3	4
	Twice the area from 0 to 3	3
	Volume of a cylinder	2
	Miscellaneous	1
All parts correct		9
All parts correct and over 50 total marks		100

Probably many of those who got the correct result in part "a" were merely guessing. A circle is the curve which most of them would think of without looking at the equation.

Part "b" is much easier than usual since it does not involve negative or fractional indices.

Question 20

(a) Sketch the graphs of the functions

(1) $x^2 - 5x + 6$

(2) $x^2 - 6x + 9$

(3) $x^2 + x + 2$

(b) Consider the equations

(4) $x^2 - 5x + 6 = 0$

(5) $x^2 - 6x + 9 = 0$

(6) $x^2 + x + 2 = 0$

What do the graphs (1), (2) and (3) tell you about the roots of the equations (4), (5) and (6)?

Solution

		%
(a)(1)	Correct	68
	Did not try it	12
	$(x - 2\frac{1}{2})(x - 2\frac{1}{2})$	8
	Very poor graph	6
	Graph of: $y = -x^2 + 5x - 6$	4
	Miscellaneous errors	2
(2)	Correct	64
	Did not try it	13
	Very poor graph	9
	Straight line graph	6
	Symmetrical to y-axis	4
	Miscellaneous errors	4

(3)	Correct	44
	Did not try it	26
	$(x + 2)(x - 1)$	5
	$(x + 1)(x + 1)$	6
	No real roots $\frac{1 \pm \sqrt{1 - 8}}{2}$ no graph	2
	Very poor graph	8
	Miscellaneous errors	9
(b)(4)	Correct	44
	Did not try it	32
	Miscellaneous errors	24
(5)	Correct	40
	Did not try it	38
	Miscellaneous errors	22
(6)	Correct	30
	Did not try it	41
	Miscellaneous errors	29
All parts correct		15
All parts correct and over 50 total marks		78

The following incorrect answers were fairly common:

"It gives minimum values."

"Where curve crosses x and y axes we get the roots of the equation."

"Equation gives you turning points, and maximum and minimum values."

"The roots tell you where the curve crosses the x axis."

"It gives the value of x when y = 0"

"Where graphs cross, the roots are real and equal."

Most of the students tried this question. Practically all of the graphs were drawn by making a table of values. Many of those who did use factors made errors in factoring.

Part "b" was answered much better than one would expect from results on previous graph questions.

CHAPTER V

Results of 1942 Examination

Question 1

If $5y = 4x$, find the value of the ratio $y : x$.

Solution	%
Correct	66
Did not try it	0
5/4	32
Miscellaneous	2
Correct and over 50 total marks	78

Question 2

$a : b = c : d$

Write a fraction equal to $\frac{a}{b}$.

Write a fraction equal to $\frac{a}{c}$.

Solution	%
Correct	78
Did not try it	0
$\frac{a}{b} = \frac{2}{3}$ and $\frac{a}{c} = \frac{1}{2}$	6
$\frac{a}{b} = \frac{ac}{bd}$ and $\frac{a}{c} = \frac{ab}{cb}$	3
$\frac{a}{b} = \frac{cb}{ad}$	3
$\frac{a}{b} = \frac{x}{y}$ and $\frac{a}{c} = \frac{x}{d}$	3
$\frac{a}{b} = \frac{1}{2}$ and $\frac{a}{c} = \frac{6}{12}$	1
Miscellaneous errors	6
Correct and over 50 total marks	82

Question 3

L varies inversely as W. How is L affected when W is halved?

Solution	%
Correct	80
Did not try it	0
L is halved	10
L gets larger	3
L is four times as large	2
Miscellaneous errors	5
Correct and over 50 total marks	68

The first three questions are quite simple ratio problems. The lower percentage correct on the first problem is probably due to the wording of the problem. In nearly all problems of this type the question is, "what is the ratio of x:y?"

Question 3 was answered better than the other two. It seems to be a more difficult problem but of course the students can mentally check their results by using concrete examples. Apparently about ten per cent do not know the meaning of "inverse ratio."

Question 1 indicates that students are not much concerned about checking their results. 32% gave the ratio as 5/4. A simple check would have shown them that this was incorrect.

Question 4

What value of x will make the function $\frac{x + 3}{5 - x}$ equal to zero?

Solution	%
Correct	68
Did not try it	1
-3 or +5	3
x = 1	12
x = 5	14
x = -5	2
Correct and over 50 total marks	83

Question 5

For what value of x is function in question 4 not defined?

Solution	%
Correct	58
Did not try it	1
-3	20
0	10
Negative values	4
-5	3
Miscellaneous errors	4
Correct and over 50 total marks	91

It is interesting to note that most of the students who could correctly solve problem "5" could do reasonably well on the rest of the paper. This problem appears to be a good one for testing the students' knowledge of algebra.

Question 6

Express the perimeter (p inches) of a semicircle as a function of its radius (r inches).

Solution	%
Correct	10
Did not try it	16
$P = \pi r$	51
$P = 2\pi r$	8
$P = 2r + 2\pi r$	5
$P = 2r$	2
$P = \pi d$	2
Miscellaneous	6
Correct and over 50 total marks	87

This problem proved to be very difficult for the students. Over half of them forgot about the diameter, or straight side, of the semicircle.

Although it did prove to be difficult I believe that this is a very good problem. The wording is clear and if the students visualize or sketch the figure it should not prove troublesome.

The students have had very little practice in formula construction and so are often afraid to try. Very little emphasis is placed on this topic anywhere in the high school course.

Question 7

If $f(x) = 5x - 2x^3$, find the value of $f(-2)$.

Solution	%
Correct	81
Did not try	2
-26	9
$-10x + 4x^3$	3
Miscellaneous errors	5
Correct and over 50 total marks	73

Question 8

As h approaches the limit 0, what limit does $\frac{5h + h^2}{h}$ approach?

Solution	%
Correct	54
Did not try it	3
Zero	29
Infinity	10
One	3
Miscellaneous	1
Correct and over 50 total marks	70

In problem 8 the difficulty with zero and infinity is again apparent. The percentage correct is somewhat the same as in question 4 and 5 which also are concerned with these two quantities.

Question 9

What is the gradient of the graph of the function $3x + 5$?

Solution	%
Correct	79
Did not try it	9
5	4
Miscellaneous	8
Correct and over 50 total marks	74

Question 10

Given $y = 5x^2 + 7$. Find $\frac{d^2y}{dx^2}$

Solution	%
Correct	84
Did not try it	0
$10 + c$	6
0	4
$\frac{5}{12}x^4 + \frac{7x^2}{2}$	2
Miscellaneous	4
Correct and over 50 total marks	76

The results on these two problems were quite good. Those who did not try problem 9 probably did not understand what had to be done. They probably could solve $\frac{dy}{dx}$ for $y = 3x + 5$.

Several of the students integrated or partially integrated in trying to find the second derivative in problem 10.

Question 11

At what value of x does x^2 change from a decreasing to an increasing function?

Solution	%
Correct	60
Did not try it	9
$+1$	14
At a minimum value	5
-1	5
All values	2
Miscellaneous	5
Correct and over 50 total marks	77

Difficulty is again experienced with the zero value.

Question 12

Find a value of x at which the slope of the graph of $x^2 - 4x$ is equal to zero.

Solution	%
Correct	59
Did not try it	2
$x = 4$	31
$x = 0$	5
$x = -2$	2
$x = -1$	1
Correct and over 50 total marks	85

The large number who gave the answer of $x = 4$ were probably thinking of the equation: $x^2 - 4x = 0$. Some students do not realize that $\frac{dy}{dx}$ will give the slope, and some are completely lost as soon as zero is mentioned.

Question 13

$\frac{dy}{dx}\Delta x$ is often used as an approximation for Δy. For which of the following values of Δx would you expect the approximation to be closed: 0.015, 0.021, 0.02?

Solution	%
Correct	54
Did not try it	3
.02	35
.021	8
Correct and over 50 total marks	73

These results are not what one would expect to get by chance but this type of question does encourage guessing. I believe this should be avoided as much as possible.

The results of all three examinations show that the students do not understand what Δy is.

Question 14

Solve $\frac{ds}{dt} = t^2$

Solution	%
Correct	36
Did not try it	4
Forgot the constant	40
2t	12
$2/3t^3$	3
2t + c	2
Miscellaneous	3
Correct and over 50 total marks	81

The two errors most commonly made in working this type of question showed quite plainly here. Students forget to add the constant when they integrate and they confuse integration with differentiation.

Question 15

Solve $\frac{ds}{dt} = \frac{1}{t^2}$

Solution	%
Correct	26
Did not try it	4
Forgot the constant but given full credit	26
$-t^{-1}$ no credit given	5
$\frac{1}{3t^3}$	6
$-\frac{2}{t^3}$	7
$\frac{1}{2t}$	5
Miscellaneous errors	21
Correct and over 50 total marks	88

As in all questions of this type a very large number of students forgot the constant which should be added. In this case all students were given full credit; the examiners apparently thinking that the penalty should not apply in both 14 and 15.

The answer $-\frac{1}{t^1}$ was given full credit but the answer $-t^{-1}$ was given, in five per cent of the cases, and received no credit. Since no mention was made of not using negative indices in answers, I believe, this answer should have been given marks.

As in question 14 a fairly large number of students differentiated when they should have integrated.

Question 16

Evaluate $\int_1^3 2x\,dx$

Solution	%
Correct	60
Did not try it	2
$2x\,dx = (x^2) - (x^2)$ for $x = 3$ and $x = 1 = 27 - 1 = 26$	8
$2x\,dx = 6 - 2 = 4$	11
Mechanical errors	10
Miscellaneous errors	9
Correct and over 50 total marks	83

Question 17

Write the fifth term in the following sequence:
1/2, 4/5, 9/10, 16/17,

Solution	%
Correct	68
Did not try it	16
Miscellaneous	16
Correct and over 50 total marks	82

This is the type of series often found on intelligence tests. If high marks are an indication of high intelligence, this test question does not seem to be a good one for intelligence test purposes.

The conscientious student may be so interested in trying to get the formula required to solve the problem; that he fails to note what the result must be, by mere inspection.

Question 18

The nth term of an A. P. is $\frac{6n + 19}{4}$. Write the 10th term.

Solution	%
Correct	61
Did not try it	21
$\frac{6n + 19}{4} = \frac{6 \times 10 + 19}{4} = 34$	7
Miscellaneous errors	11
Correct and over 50 total marks	84

This is certainly an easy problem but apparently students were looking for difficulties which did not exist. Students often do not really know what is meant by the nth term. They do not seem to be able to change from the general to the specific term.

Question 19

The first two terms of a G. P. are 15 and 10. Write the third term.

Solution	%
Correct	72
Did not try it	1
5	15
Miscellaneous errors	12
Correct and over 50 total marks	79

It is surprising to find more correct answers for this problem than for the previous one. Apparently the students know the formula to use for geometric progression.

Question 20

Given $f(x) = (1.03)^x$. Use tables to find the values of f(21) correct to the second decimal place.

Solution	%
Correct	20
Did not try it	17
1.857	28
Less than one	8
Over fifteen	7
Other answers	20
All correct and over 50 total marks	99

Those who gave the answer 1.857 must have used logarithms to get their answer. This answer is correct by logarithms to three decimal places, the last figure being of course doubtful. Students do not make use of the Amount Table which is the most convenient method of solving problems of this type. The students failed to heed the wording of the question, "correct to the second decimal place."

15% had wholly unreasonable results. This shows that students fail to check their results with any degree of care.

Question 21

How many different committees consisting of one man and one woman can be chosen from a group of 5 men and 6 women?

Solution	%
Correct	74
Did not try it	7
$5C_1 \times 6C_1$	4
Miscellaneous errors	15
Correct and over 50 total marks	68

Since the number of students getting the correct result was high, and the percentage of those having correct results and high total marks was low, it would seem that this question is a poor test of the students ability. Many students probably solved the problem by doing the most obvious thing with the two numbers given.

Question 22

Evaluate $\frac{10!}{6!\,4!}$

Solution	%
Correct	77
Did not try it	5
1	5
Miscellaneous errors	13
Correct and over 50 total marks	67

Question 23

If $(x + 5)(2x + 3)(3x + 2)(5x + 1)$ is expanded and arranged in descending powers of x, what are the first and the last terms?

Solution	%
First Part	
Correct	80
Did not try it	6
$5x + 1$	9
Miscellaneous	5
Second Part	
Correct	70
Did not try it	6
$x + 5$	9
11	5
Miscellaneous	10
Both parts correct and over 50 total marks	73

Question 24

Using a table of logarithms, solve for x : $10^x = 37.5$

Solution	%
Correct	26
Did not try it	37
3.75	7
Error in decimal point	5
.1574	4
.574	3
5.74	2
Miscellaneous errors	16
Correct and over 50 total marks	84

Those who gave the answer as 3.75 apparently read the question as 10x = 37.5, solve for x. Those giving answers involving the figures 574 looked up the logarithm of 37.5 but did not know what to do with it. That is, they did not realize that logarithms are to base 10 and that all that is asked is, "What is the logarithm of 37.5 to base 10?"

Question 25

What is the gradient of the line joining the points (2,1) and (5,6)?

Solution	%
Correct	71
Did not try it	7
1 1/3	9
3/5	3
A straight line	2
Miscellaneous errors	8
All correct and over 50 total marks	80

Question 26

The ratio of the volumes of two circular cylinders is 15:21. Find the ratio of their diameters.

Solution	%
Correct	24
Did not try it	25
9/2	17
2:1	6
Miscellaneous	28
Correct and over 50 total marks	83

Those who gave 9/2 as an answer took $\frac{V_1}{V_2} = \frac{d_1h_1}{d_2h_2}$.

Those who gave 2:1 as their answer multiplied 15/5 by 14/21 and since it gave a simple ratio they assumed it was correct.

The result is about the same as in 1941 when 26% obtained the correct result on a similar problem.

Question 27

The time of oscillation of a pendulum varies as the square root of its length. A pendulum 24 inches long makes one complete oscillation in $1\frac{1}{2}$ seconds. Calculate to the nearest hundredth of an inch the length of a pendulum which makes one complete oscillation in one second.

Solution	%
Correct	19
Did not try it	9
10.66	14
10.63	10
10.69	9
Mechanical errors	21
Miscellaneous errors	18
Correct and over 50 total marks	81

About one third of the students obtained answers correct to the nearest tenth but lost marks because the answer had to be given correct to the nearest hundredth.

Mechanical errors were very high. Some of these gave very bad results: as low as .0306 and as high as 32.66.

Question 28

When $s = 2$, $V = \frac{1}{\sqrt{t}}$ and when $t = 4$, $V = \frac{s^2}{8}$. Derive a formula expressing V in terms of s and t.

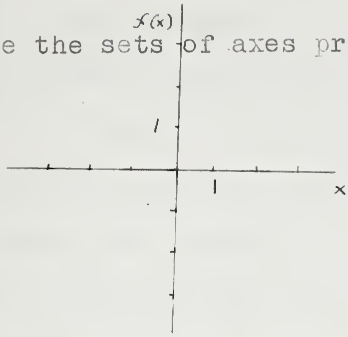
Solution	%
Correct	15
Did not try it	47
$V = \frac{Ks^2}{8\sqrt{t}}$	11
$V = \frac{1}{2}\frac{s^2}{\sqrt{t}}$	5
Miscellaneous	22
Correct and over 50 total marks	100

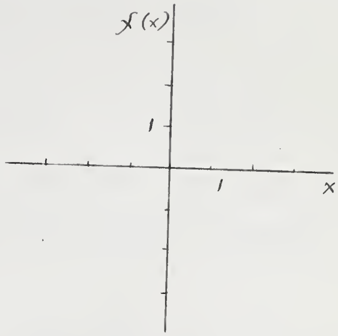
This problem proved to be quite difficult for most of the students. Those who did solve it did not have much trouble with the rest of the paper. It was a good problem for distinguishing between strong and weak students.

Question 29

Sketch the graph of (a) $\frac{x + 1}{x - 1}$ and (b) $\frac{x + 1}{1 - x}$

Use the sets of axes provided below.

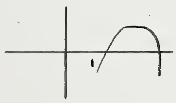




Solution %

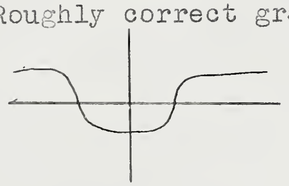
(a) Correct 40

Did not try it 5



6

Roughly correct graph no asymptotes 6



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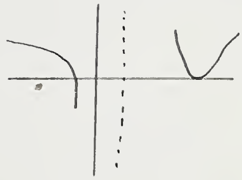


5

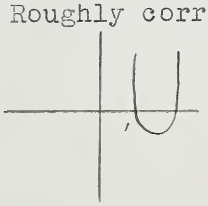
Part of the curve correct 6

Miscellaneous errors 27

(b) Correct 32

Did not try it 13

Roughly correct graph no asymptotes 10



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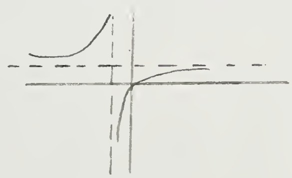


5

One part of curve correct	4
Miscellaneous errors	30
All parts correct	24
All parts correct and over 50 total marks	95

This is a fairly difficult graph and it was reasonably well done by the students. A very small percentage avoided the question. The fact that the axes were drawn and named and that the scale was shown probably encouraged many to try the graph.

Most of the errors were made where $x \longrightarrow 1$. The concept of infinity seems to be very difficult for the students.

Question 30

A right triangle has a perimeter of x inches, and one of the sides about the right angle is 4 inches long. Express the area of the triangle (A sq. in.) as a function of x.

Solution	%
Correct	0
Did not try it	48
$y = \frac{8x - x^2}{8 - 2x}$	1
$2x - 8$	5
$x - 8$	4
Miscellaneous	42

Only one student solved this question correctly. The problem is fairly difficult but the wording is clear and the problem should be handled readily by the better Grade Twelve students.

A few of the students solved the other two sides of the triangle but then failed to give the area. They were either very doubtful about their result or failed to note what was required.

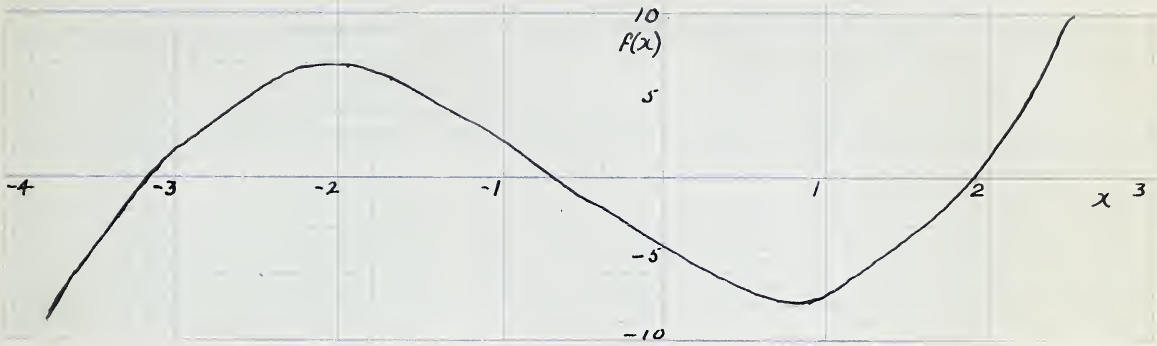
The errors were so diverse that I could see no value in listing them. $2x - 8$ and $x - 8$ were the two answers given most often. Neither of these show much thought of the real problem on the part of the student.

Question 31

The diagram below shows the graph of $x^3 + 2x^2 - 5x - 4$.

Use this graph to solve the equation $x^3 + 2x^2 - 5x - 4 = 0$.

Give answers to the nearest tenth.



Solution	%
Correct	40
Did not try it	27
Answers to nearest hundredth	12
One correct root given	3
Two correct roots given	2
Miscellaneous	16
Correct and over 50 total marks	94

This problem is very easy if the students know how to read a graph. The results show that many of the students cannot read graphs.

12% of the students were given partial credit for their answers because they gave results to the nearest hundredth. They read the graph correctly but failed to note the instructions in the problem.

Question 32

From the graph given in question 31, solve the equation $x^3 + 2x^2 - 5x + 1 = 0$. Give answers to the nearest tenth.

Solution	%
Correct	13
Did not try it	39
Miscellaneous	48
Correct and over 50 total marks	100

As shown again here, the students do not know what a graph tells them. They can draw a graph but cannot interpret it.

There was no agreement in the errors. Most of the students drew another graph, by taking various values of x and plotting the points found. Even when this was done correctly they still did not know how to find the solution of the problem.

Question 33

On the set of axes used for the graph in question 31, draw the graph of $2x - 1$. Show values used for plotting in the table provided below.

x	
$2x + 1$	

Solution	%
Correct	85
Did not try it	0
$y = 2x - 1$	6
Miscellaneous errors	9
Correct and over 50 total marks	78

Students do not have much difficulty in drawing a simple graph. After it is drawn however, it proves to be of little value to them since they do not know how to use it.

Question 34

Using the original graph given in question 31 and the graph which you were directed to draw in question 33, solve the equation $x^3 + 2x^2 - 7x - 5 = 0$. Give answers to the nearest tenth.

Solution	%
Correct	21
Did not try it	50
Misread one or two points	5
Miscellaneous errors	24
Correct and over 50 total marks	98

85% of the students had question 33 correct but only 21% of the students could read the correct solution. Students certainly need more training in learning how to interpret graphs.

There were several slips on the part of the examiners on this question. Several students gave results of 2.2, -.6, - 3.7; which are the correct results but given in reverse order. These were marked incorrect.

Question 35

Consider functions of the form $ax^2 + bx + c$. How are the graphs of such functions restricted in shape or position when $a \neq 0$, $b \neq 0$, $c \neq 0$?

Solution	%
Correct	22
Did not try it	31
Straight line graph	32
Graphs are not restricted	4
Miscellaneous	11
Correct and total marks over 50	90

Question 36

As in question 35, how are the graphs restricted when $a \neq 0$, $b = 0$, $c \neq 0$?

Solution	%
Correct	6
Did not try it	37
Graph is a curved line	17
Graph would be a parabola with vertex at C	16
Curve doesn't pass through origin	8
When $C \neq$ the line cuts the y axis above or below the origin	7
Graph goes down as a straight line	6
Miscellaneous errors	3
Correct and over 50 total marks	100

Question 37

As in question 35, how are the graphs restricted when $a \neq 0$, $b \neq 0$, $c = 0$?

Solution

	%
Correct	4
Did not try it	38
It passes through origin	26
Graph is a parabola	10
Graph does not touch either y axis or x axis	7
Graph is an ellipse	5
Graph coincides with x axis	2
Graph has a definite slope	1
Graph is a hyperbola	1
Miscellaneous	6
Correct and over 50 total marks	100

These three problems are similar. The first one was best understood by the students as it is a straight line.

The students generally have some idea of the correct graph but are very vague about it. A rough sketch would have helped the students to explain what they meant but none of them thought of doing this.

Question 38

For what range of values of K are the roots of the equation $2x^2 - 3x + K = 0$ real and unequal?

Solution	%
Correct	1
Did not try it	33
$0 < K < \frac{9}{8}$	17
$K = -2$	7
$K < \frac{3}{4}$	6
$9 - 8K$	6
$K = \frac{1}{2}$	3
K must be negative	3
$b^2 = 9$	2
$0 > K > -\infty$	2
Miscellaneous	20
Correct and over 50 total marks	100

Probably a fairly large number of students could define "discriminant" and tell what the roots would be when the discriminant is greater than zero. That is, students memorize certain things but do not really understand them and so are unable to apply that which they have memorized.

Question 39

The function $2x^2 + x - 1$ may be expressed in the form $2(x + A)(x + B)$. Find A and B.

Solution	%
Correct	34
Did not try it	25
$A = -1$, $B = 1$	10
$A = -\frac{1}{2}$, $B = \frac{1}{2}$	7
$A = -1$, $B = \frac{1}{2}$	6
$A = -\frac{1}{4}$, $B = \frac{1}{4}$	2
Miscellaneous errors	16
Correct and over 50 total marks	83

This is a problem which should not be too difficult for anyone who has taken Algebra 1. Part of the difficulty is probably due to the fact that students often take Algebra 1 in grade ten and then take Algebra 2 in grade twelve. This means that the students often forget a great deal that was learned in the first algebra course by the time they start the second course.

Question 40

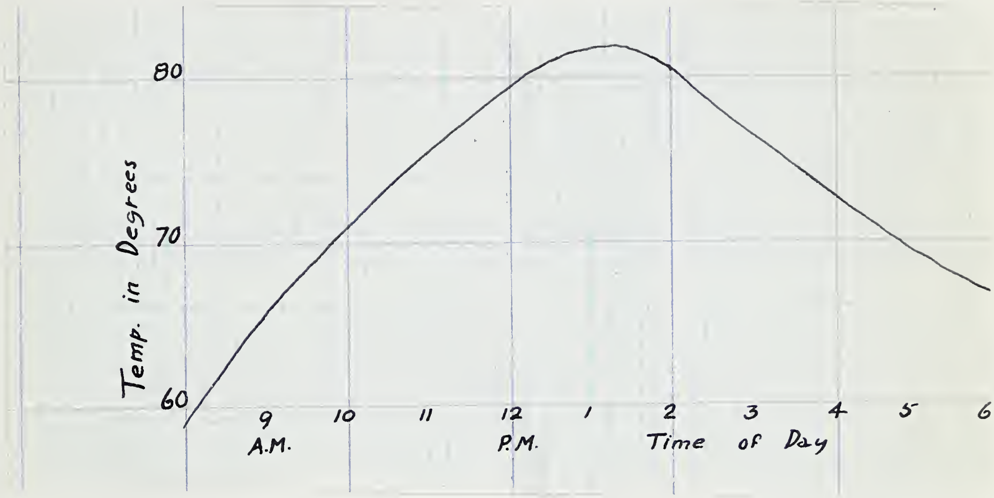
As n increases without limit, the functions $\frac{n^2}{n^2+1}$ and $\frac{n^2}{n^2-1}$ both approach the limit -1. How do they differ in their manner of approaching the limit?

Solution	%
Correct	32
Did not try it	19
1st function large, 2nd function small	12
1st function approaches 1 from positive side 2nd function approaches 1 from negative side	21
1st approaches 1 from right side 2nd approaches 1 from left side	10
Miscellaneous	6
Correct and over 50 total marks	81

The students did not draw a rough sketch of the graph to aid them in answering the question. Without this aid they fail to visualize what the graph would look like and so are hazy in their descriptions of it.

Question 41

The accompanying diagram shows the variations in temperature at a certain place during part of a summer day. Find the average rate of change of the temperature between 11 a. m. and 2 p. m.



Solution	%
Correct	14
Did not try it	17
Just gave 2° per hour did not state a rise of 2° per hour	29
Practically correct, partial ~~correct~~ credit	5
77°	4
Miscellaneous errors	31
Correct and over 50 total marks	79

Question 42

From the diagram given in question 41, estimate the rate of change of temperature at 1:30 p.m.

Solution	%
Correct	9
Did not try it	34
81.5	7
Approximately correct	4
$-\frac{4}{5}$	4
Error in sign	7
Miscellaneous errors	35
Correct and over 50 total marks	96

In question 41 and 42 we find that the students cannot read graphs. In question 41 many of them lost marks for not stating that the change was a rise. Probably many of them thought this was so obvious it was not necessary to state it.

Question 45

If $y^2 = x^3$, find the value of $\frac{dy}{dx}$ when $x = 4$.

Solution	%
Correct	7
Did not try	6
$\frac{dy}{dx} = 3$ (forgot $\pm$)	40
$y^2 = x^3 \quad \frac{dy}{dx} = 3x^2 = 48$	15
$y^2 = x^3 \; ; \; 2y = 3x^2 \; ; \; y = 24$	7
$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2\sqrt{x}} = \frac{3}{4}$	6
Miscellaneous errors	19
Correct and over 50 total marks	86

A large number of students forgot to state the sign in their answer. They were given three marks out of four in this case which I think was quite fair.

15% of the students just ignored the index in y^2. Some probably did not notice it and some probably did not know what to do with it.

We again find students who do not understand how to use fractional indices. Negative and fractional indices should be given more attention in school.

Question 44

Find all the maximum and minimum values of $4x^3 + 15x^2 - 18x + 7$.

Solution	%
Correct	1
Did not try it	33
$x = -3$ or $+\frac{1}{2}$	34
One part correct	5
Mechanical errors	11
Miscellaneous errors	16
Correct and over 50 total marks	100

One-third of the students found the values of x at which there was a maximum or a minimum but they did not calculate the maximum and minimum values. There was some variation in marking this result. In most cases it was given two marks but in others it was given four marks. I believe that four marks is a fairer value when one considers what part of the work has been done correctly.

Question 45

The volume of a sphere is $4/3\pi r^3$. If the radius of the sphere is increased from 5 cm. to 5.03 cm. use the theorem mentioned in question 13 to obtain an approximate value for the increase in the volume of the sphere. Give the answer correct to two decimal places.

Solution	%
Correct	23
Did not try it	35
3π	8
Mechanical error	6
Miscellaneous errors	28
Correct and over 50 total marks	79

Question 46

Given $\frac{d^2s}{dt^2} = 3t + \frac{1}{2}$. Also, when $t = 2$, $s = 6$ and $\frac{ds}{dt} = 10$.

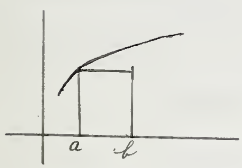
Express s as a function of t.

Solution	%
Correct	18
Did not try it	41
$\frac{ds}{dt} = 3/2\, t^2 + \frac{1}{2}t + c$	17
Forgot the constant	11
Mechanical errors	8
Miscellaneous errors	5
Correct and over 50 total marks	98

Question 47

Using the sketch below, mark the boundary of the area $\int_a^b f(x)\,dx$ with four heavy lines.

Solution	%
Correct	37
Did not try it	24
Did not show "a" or "b" but had correct drawing	8



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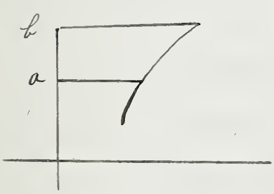

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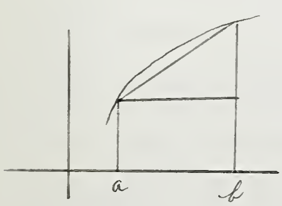

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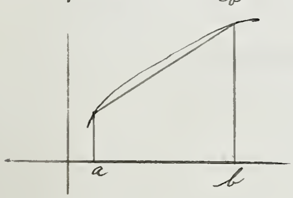

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Miscellaneous	8
Correct and over 50 total marks	96

The wording of this question tended to confuse the students. Many of them probably knew the correct area, but when asked to mark it with four lines; the found they had two lines extra, so they put them in some place.

It would have been better to say "shade the area" instead of asking for it to be shown by four heavy lines.

Question 48

A sphere having a diameter of 10 inches is divided into two parts by a plane which is 3 inches from the centre of the sphere. Use integration to find the volume (correct to one decimal) of the smaller part.

Solution	%
Correct	0
Did not try it	48
17 1/3	3
Approximately correct	2
Miscellaneous errors	47

This problem proved to be too difficult for any of the students. Most of the students could not construct the formula and set up the integral. Those who did succeed in doing this made errors in integration or made mechanical errors. Either this section has been presented badly or it is too advanced for high school students.

Question 49

The first term of an A. P. is 3 and the seventeenth term is -27. Find the sum of these seventeen terms.

Solution	%
Correct	61
Did not try it	8
Incorrect formula	7
Mechanical error	8
Miscellaneous errors	16
Correct and over 50 total marks	78

Question 50

If the sum of the first n terms of the A. P. in question 49 is 486, find n.

Solution	%
Correct	1
Did not try it	24
$S = 2a + (n - 1)d$; $n = 32.03$	12
$S = \frac{n}{2}\left[6 + (n - 1)3/2\right]$ Did not solve this	12
$S = \frac{n}{2}(a + l) = \frac{n}{2}(3 + 27) = 32.4$	11
$486 = 3 + \frac{3}{2}(n - 1)$ $n = 323$	7
Errors in factoring	15
Miscellaneous errors	18
Correct and over 50 total marks	100

Most of the students used the wrong formula. Those who did use the correct formula made some other error. The students seems to be very weak in factoring. This is probably due to lack of practice for a considerable time.

Question 51

Find (to the nearest cent) the interest earned by investing \$250.00 for 7 years at 3% per annum compounded semi-annually.

Solution	%
Correct	8
Did not try it	21
$250 \times (1.03)^{14}$	8
$250 \times (1.03)^{7}$	4
Attempted the annuity formulae	23
Miscellaneous	36
Correct and over 50 total marks	95

Students who have taken grade twelve mathematics should be able to solve a problem like this. As a matter of fact, it used to be done in grade nine arithmetic and did not prove too difficult then.

I believe that the section of the text on annuities, perpetuities, and capitalization, confuse the students so much; that they fail to solve simple problems like the one given here.

This topic could be presented in a much simpler form. Apparently teachers have not shown the students the easier method of solving problems of this type. The text gives a method and the instructors follow this method.

Question 52

A \$50.00 bond bearing interest at 3% is redeemable at par in 10 years. Find to the nearest cent the cash value of this bond if the current discount is $2\frac{1}{2}$%.

Solution	%
Correct	2
Did not try it	43
$\frac{50}{(1.025)^{10}}$	17
$\frac{50(1.03)^{10}}{(1.025)^{10}}$	6
Miscellaneous errors	32
Correct and over 50 total marks	83

The results show that this section is too difficult for the students or this section of the text is not being taught properly.

Question 53

There are 17 different books from which the winner of a contest may select the 3 to which he is entitled. Of these 17, 10 are fiction and 7 are non-fiction. If at least one of the books chosen must be non-fiction, in how many ways can the three books be selected.

Solution	%
Correct	18
Did not try it	10
$10C_2 \times 7C_1 = 315$	27
Mechanical error	9
$10C_2 \times 7C_3 = 1575$	4
Permutation	9
Miscellaneous errors	23
Correct and over 50 total marks	97

Some of the difficulty in this problem is due to failure to read the question carefully. 27% gave the answer based on choosing one non-fiction book. The students take "at least one" to mean merely "one".

While this difficulty arises in all problems it causes more trouble in this section of the book than in any other. Students must learn to read carefully in order to see what is given and what is required.

Question 54

Write down and simplify the first four terms in the expansion of $(x - \frac{y}{2})^{16}$.

Solution	%
Correct	34
Did not try it	14
Error in working combinations	23
Error in signs	11
Error in indices	5
Miscellaneous errors	13
Correct and over 50 total marks	93

Question 55

Write the middle term in the expansion of the binomial named in question 54.

When $x = 2$ and $y = 1$, what is the value of this term?

Solution	%
Correct	16
Did not try it	18
Mechanical error only	15
Wrong term	9
Miscellaneous errors	42
Correct and over 50 total marks	87

In question 55 the errors were of all types. 15% made a mechanical error only but the total per cent of mechanical errors was very high.

Question 56

Assuming that, under certain conditions, $(1 + x)^n \simeq 1 + nx$, find an approximate value for $\frac{1}{\sqrt{104}}$.

Solution	%
Correct	11
Did not try it	12
$(1 + .04)^{-\frac{1}{2}} = .98$	9
$(100 + 4)^{-\frac{1}{2}} = 98$	8
$100\ (1 + .04)^{-\frac{1}{2}} = 100(.98) = 98$	6
$\frac{1}{10}\ (1 + .04)^{-\frac{1}{2}} = \frac{1}{10}\ (1 - .08) = .092$	4
Miscellaneous errors	50
Correct and over 50 total marks	68

Most of the students do not know how to take out a perfect square or perfect cube from under the root sign. Indices cause considerable difficulty. Mechanical errors are very common.

CHAPTER VI

Summary of Results

Table II

Allotment of Marks

Section of Text	Pages in Text	Marks 1940	1941	1942
Ratio and Variation	22	15	15	19
Functions of one Variable	20	22	18	45
Limits and Gradients	17	10	10	16
Differentiation	23	32	35	24
Integration	10	11	19	18
Sequences and Series	22	22	18	23
Permutations and Combinations	11	5	9	9
Binomial Theorem	10	9	5	9
Empirical Formulae	9	8	10	
	Total	134	139	163

The table above shows that the examinations were excellent, in allotment of marks for each section of the text. The problems on the examination seem to be fair samples of those in the text in regard to difficulty.

Each question was classified as to its topic. Then the average percentage correct for every problem from a section was found. Some of the problems in certain sections were relatively easy and some were relatively difficult. It must be admitted that the final average obtained therefore is not an exact picture but they probably show the relative difficulty of the sections. The percentages correct do not

include those problems where partial credit was given.

Table III

Average Results for Different Sections of Text

Section of Text	Percentage Correct	Percentage Did not attempt problem
Ratio, Proportion, and Variation	36	19
Functions of One Variable	37	28
Limits and Gradients	38	40
Differentiation	41	22
Integration	40	23
Sequences and Series	28	36
Permutations and Combinations	31	14
Binomial Theorem	33	40
Empirical Formulae	20	53

Table IV

Results Each Year by Sections

Ratio, Proportion and Variation

1940			1941			1942		
Item No.	% Correct	Not Attempted	Item	% Correct	Not Attempted	Item No.	% Correct	Not Attempted
1	22	10	1 (c)	87	0	1	66	0
3	7	45	3	21	22	2	78	0
6	16	12	4	32	36	3	80	0
			5	44	31	26	24	25
						27	19	9
						28	15	47
Functions of One Variable								
8 (a)	75	10	1 (d)	48	10	4	68	1
(b)	56	12	(e)	62	5	6	10	16
(c)	49	18	(f)	87	9	7	81	2
(d)	52	18	6 (a)	12	65	20	20	17
(e)	36	16	(b)	10	67	24	26	37
17 (a)	29	34	(c)	3	79	29 (a)	40	5
(b)	27	38	13 (a)	54	35	(b)	32	13
(c)	33	51	(b)	46	46	30	0	48
(d)	1	69	(c)	30	57	31	40	27
(e)	1	76	(d)	8	74	32	13	39
						33	85	0
						34	21	50
						35	22	31
						36	6	37
						37	4	38
						38	1	33
						39	34	25

Limits and Gradients

1940			1941			1942		
	%	Not		%	Not		%	Not
Item No.	Correct	Attempted	Item	Correct	Attempted	Item No.	Correct	Attempted
11 (a)	22	56	1 (g)	63	6	5	58	1
(b)	12	77	(h)	22	10	8	54	3
(c)	17	71	20			11	60	9
(d)	7	81	(a) 1	68	12	25	71	7
(e)	5	87	2	64	13	40	32	19
(f)	5	86	3	44	26	41	14	17
(g)	6	88	(b) 1	44	32	42	9	34
			2	40	38			
			3	30	41			

Differentiation

1940			1941			1942		
Item No.	% Correct	Not Attempted	Item	% Correct	Not Attempted	Item No.	% Correct	Not Attempted
5 (a)	95	2	1 (a)	92	0	9	79	9
(b)	96	2	(i)	49	26	10	84	0
(c) 1	42	10	(j)	39	18	12	59	2
2	51	21	2 (a)	28	24	13	54	3
3	29	26	(b)	81	16	43	7	6
(d)	44	25	(c)	12	46	44	1	33
(e)	30	43	4 (a)	30	39	45	23	35
7 (a)	45	5	(b)	34	32	56	11	12
(b)	40	5	7 (a)	14	47			
9 (a)	13	35	(b)	32	49			
(b)	13	47	(c)	14	46			
(c)	21	16	9 (a)	16	50			
			(b)	30	48			
			(c)	14	58			
			17 (a)	34	15			
			(b)	22	68			
			(c)	14	72			
			(d)	5	66			

Integration

1940			1941			1942		
2 (graph)	65	14	1 (b)	24	4	14	36	4
(Intes.)	42	14	8 (a)	34	30	15	26	4
12 (a)	54	22	(b)	17	39	16	60	2
(b)	24	39	11 (a)	58	18	46	18	41
			(b)	30	47	47	37	24
			19 (a)	22	59	48	0	48
			(b)	14	48			
			(c)	12	78			

Sequences and Series

	1940 %	Not		1941 %	Not		1942 %	Not
Item No.	Correct	Attempted	Item	Correct	Attempted	Item No.	Correct	Attempted
13 (a)	66	9	15 (a)	66	24	17	68	16
(b)	15	56	(b)	5	77	18	61	21
(c)	76	19	18 (a)	5	56	49	61	8
(d)	26	31	(b)	1	73	50	1	24
15 (a)	6	33				51	8	21
(b)	25	35				52	2	43
(c)	14	46						

Permutations and Combinations

4	17	16	10 (a)	18	18	21	74	7
			(b)	26	24	22	77	5
			(c)	16	34	53	18	10

The Binomial Theorem

14 (a)	25	42	12 (a)	44	42	33	85	0
(b)	0	72	(b)	46	39	54	34	14
(c)	3	89	(c)	8	56	55	16	18
(d)	9	53						

Empirical Formulae

16 (a)	10	65	16 (a)	48	20
(b)	1	59	(b)	46	32
			(c)	43	29
			(d)	21	42
			(e)	20	57
			(f)	19	65

The percentage of the problems not attempted was much lower on the 1942 examination than on the two previous examinations. This paper was criticized for its length but apparently the students had time enough to complete it.

Table V

Comparison of Results on Problems

of

Approximately Equal Difficulty

on the

Three Papers

	Percentage Correct		
	1940	1941	1942
Simple variation		87	80
Simple problem on limits	52	48	58
Ratio of volumes of similar objects	25	21	24
Problem on combinations	17	18	18
Simple differentiation	95	92	84
Average type differentiation	45	34	
Difficult differentiation	13		7
Approximation by rule		14	11
Simple integration	24	34	26
Simple example on binomial theorem	42	44	34
Cash value of bond	6	5	2
Expressing an area as a function of x		12	0
Difficult variation problem	7	15	15

For every problem I found the percentage of those who had the problem correct and who had secured over fifty total marks. From this, I had hoped to find which problems served best, for selecting the better students.

The results do indicate that certain types of problems are much better than others for testing the general ability of students. The extremely small number, in some cases, who did have the correct answer; makes any general conclusions unwarranted.

CHAPTER VII

COMMENTS

The 1942 paper had fifty-six questions on it. The table of results show that a much higher percentage of the problems were attempted than on the two previous papers. This would indicate that the paper was not too long.

The 1942 paper eliminates trailing error which shows up so much on the first two examinations. This is the chief advantage of this type of examination.

The results obtained on problems of approximately the same type, as shown in the table in the summary, indicate that the quality of the students for the three years remains about the same.

The marking in the 1941 paper was not good. On this examination there were too many careless mistakes made by the markers and there seemed to be a poor system used for allowing partial credit for questions.

Special Weaknesses As Shown by Results

Class A

All of these reflect the prior training of the student as well as current training.

Arithmetical operations

Fractional and negative indices

Visualization of relationships

Interpretation of formulae

Interpretation of graphs

Failure to check results

Class B

Special points of difficulty in the course.

Understanding technical terms -- continued proportion

Comparison of similar figures or objects according to principles of proportion

Finding the increment ΔY of a function

The binomial approximation

Differentiation with respect to three variables

Forgetting the constant in integrating

Construction of formulae

Class C

Topics which seem to offer difficulty

Annuities

Variation

Binomial theory

The above difficulties are the principal causes of the poor results on the last three examinations. aving found the cause can we find a cure?

The most outstanding weakness is in simple arithmetical calculations. This is especially apparent in multiplication. Where problems were given which required multiplication the average error in this alone was about twelve per cent.

Several remedies are available. One, is to insist on a higher standard in the lower grades. A second, is to take short periods for practice in the high school. A third one, and one that should be insisted on at all times, is to check the result. One of the simplest and fastest methods is to obtain the approximate answer by inspection and compare the final answer with it.

As an illustration of the importance of this point let us consider Air Navigation. Our students fail these examinations, and, what is much worse, lose their lives; not because the subject is so difficult that they cannot understand it, but because they cannot add, subtract, multiply and divide. They never learned to check their results in school and it is hard to break old habits. They put down a wind of 180/40K when it should be 360/40K and end up in the sea when a simple check of their result would have shown them their error.

Teachers and examiners are at fault for allowing nearly full marks for these so-called mechanical errors. Rightfully enough, perhaps, when it is in the second or third decimal place but fatal practice when ridiculous answers are given nearly full credit.

Another point in connection with this is the failure to allow credit for an answer obtained from a graph. Often in practice, an approximate answer as found from a graph is all that is required; yet this is discouraged in every way.

Teachers and students have apparently paid little attention to the excellent treatment given in the text for finding Δy and showing what it means. In all problems where Δy was desired the percentage correct was very small but nearly all of the students could find $\frac{dy}{dx}$. Students should certainly understand what they are trying to do before rushing ahead with a method which will give a correct result but may be meaningless.

Students have learned how to draw graphs but they have not learned how to use them. This type of learning is of little value. Somewhere in the high school course, students should really learn how to use graphs. Then, having learned how to use them; they should be encouraged to use them at every opportunity. It is not laboriously drawn, exact, graphs that we should demand; but quick, rough, sketches which give us a picture of the problem.

Get an approximate answer from a sketch; and then, work out the problem; should be the general procedure in many cases.

The difficulties listed in Class A are chiefly problems for the lower grades. No teaching of advanced principles can give good results when the elementary principles are not understood.

The difficulties listed in Class B are problems for the individual instructor. In some cases a simpler approach to the problem will enable the student to understand it. In other cases more definite instruction is required. In all cases more practice in working simple problems, in which the principle is involved, should be carried out.

The difficulties listed in Class C are in a different category. I believe that the text itself is largely to blame here. The topics are either too difficult for students at this level or the treatment of the topics in the text is not satisfactory.

The Text

The results have been bad and the first thing to be blamed is the text. Let us consider it.

Algebra 2 is only required by those students who intend to go to university. For all others it is an option which they may take if they wish; the only requirement being, that they have credit in Algebra 1.

The text has been prepared primarily for those who are comparatively good in mathematics. It is intended, I believe, to test the students' ability to do University work. For this purpose I believe it is good. For the average Grade twelve student, however, I believe it is not satisfactory.

If we can find some method to prevent those students who are not fitted for it, from taking Algebra 2, then the text is excellent. If we cannot, then the harm done to these students is too great to balance the good. The weak student needs to build up confidence and confidence is destroyed when students attempt that which is completely beyond them.

One section of the text should be omitted for all students or else treated much more fully. This is the section on Annuities and Perpetuities. Probably 95% of the students leave this section of the book in a thorough muddle. They are so completely bewildered, that they even forget what they once knew. Omit this section and one of the big difficulties in this course will disappear.

HIGH SCHOOL AND UNIVERSITY MATRICULATION EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1940

ALGEBRA 2

Time—3 hours.

Note—It is not expected that any candidate will complete the paper. The candidate may work the questions in any order.

Graph paper and logarithmic tables will be supplied by the presiding examiner.

Values

1. If y varies as x^2 what is the effect

1 (*a*) on y of dividing x by 9;

2 (*b*) on x of dividing y by 9?

6 **2.** Sketch the graph of $y = 5x - 4 - x^2$ and find the area between the x-axis and the portion of the curve above the x-axis.

6 **3.** The weights of two spheres are in the ratio 20 : 9 and the densities of the materials of which they are made are in the ratio 5 : 18. What is the ratio of the radii?

5 **4.** The enrolment in a certain school is as shown in the table.

	Grade XI	*Grade XII*
Boys	16	14
Girls	18	9

In how many ways may a committee of six be chosen if there are to be on the committee two boys and one girl from Grade XII, and one boy and two girls from Grade XI?

5. Given $y = x^2$.

1 (*a*) Find $\frac{dy}{dx}$ for this function.

1 (*b*) When $x = 1.5$, evaluate $\frac{dy}{dx}$.

(*c*) When $x = 1.5$ and $\triangle x = 0.2$,

2 (1) find $\triangle y$;

1 (2) find $\frac{dy}{dx} \triangle x$;

1 (3) find $\triangle y - \frac{dy}{dx} \triangle x$.

2 (*d*) What is the average gradient of the curve between $x = 1.5$ and $x = 1.7$?

1 (*e*) What is the gradient of the curve at (1.5, 2.25)?

6 **6.** Given $x : y = 2 : 5$ and $y : z = 4 : 3$ and $z : w = 4 : 5$ express in its simplest form $x : y : z : w$.

2 **7.** If $y = \frac{4}{x^3} + \frac{6}{\sqrt{x}}$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, expressing the results without

3 negative indices.

Values

8. A piece of wire 20 inches long is bent so as to form a rectangle. One side of the rectangle is x inches and the area of the rectangle is A square inches.

3 (*a*) Express A as a function of x.

2 (*b*) Graph A against x.

2 (*c*) Find the range of values of x for which A is an increasing function of x.

1 (*d*) Find the range for which A is decreasing.

2 (*e*) Find the value of x for which A is a maximum.

4 **9.** (*a*) Given $y = z^4$ and $z = x^3 - 5x^2 + 4$, find $\frac{dy}{dx}$ in terms of x.

1 (*b*) Use the method of (*a*) to find $\frac{d}{dx}(3x + 1)^2$.

1 (*c*) Use some other method to find $\frac{d}{dx}(3x + 1)^2$.

10. $y = 2x^3 - 15x^2 + 36x$.

3 (*a*) Find the co-ordinates of each of the points on the curve at which the tangent is horizontal.

2 (*b*) Find *one* point at which the tangent has negative slope.

2 (*c*) Find the co-ordinates of a minimum point.

1 (*d*) How do you know that the point in (*c*) is a minimum point?

1 (*e*) Find the co-ordinates of a point which is on the curve and also on the x-axis.

3 (*f*) Draw a freehand sketch of the curve.

11. The tangent to the curve $y = \frac{100}{x}$ at the point C (5,20) meets the x-axis at A and the y-axis at B; P is any point on the curve.

2 (*a*) Sketch the graph of the curve.

1 (*b*) What is the gradient of the tangent at $P(a, \frac{100}{a})$?

1 (*c*) What is the gradient of the tangent at C?

1 (*d*) What is the gradient of the straight line through C and A?

2 (*e*) What is the equation of the tangent to the curve at C?

1 (*f*) What are the co-ordinates of A?

2 (*g*) What is the area of $\triangle AOB$?

3 **12.** (*a*) Solve $\frac{dy}{dx} = x^5 + 2x^2 - 3$.

2 (*b*) What is the solution of (*a*) for which $y = 5$ when $x = 2$?

3 **13.** (*a*) What is the average of the numbers in the series 8, 11, 14, 17, 20,..341, 344?

3 (*b*) If t_n is the n^{th} term of the series in (*a*) above, express t_n as a linear function of n, i.e. $t_n = (?) + (?)n$.

3 (*c*) Using a (first term), l (last term) and n, find an expression for S_n the sum of n terms of an arithmetic series.

2 (*d*) Use (*c*) to find the sum of the series in (*a*).

Values

1 **14.** (*a*) How many terms are there in the expansion of $(1 + x)^{2n-1}$, if n is a positive integer?

4 (*b*) Use two terms of a binomial expansion to find an approximate value of $\sqrt[3]{1729}$.

2 (*c*) By what per cent does this cube root exceed $\sqrt[3]{1728}$?

2 (*d*) Write the fifth term of the expansion of $(1 - x)^{\frac{1}{2}}$.

15. Deduce from the appropriate series the formula for each of the following. Do not evaluate.

3 (*a*) The amount of \$400 in 3 years with interest at 5 per cent per annum, compounded half-yearly.

4 (*b*) The cash value of an annuity of \$2000 running 20 years, first payment due six years from now, money worth 5%.

4 (*c*) The annuity, first payment due one year hence, which at 3% will accumulate to \$5000 ten years from now.

16. Given four points representing pairs of observations: (1, 88), (2, 41), (4, 16) and (5, 19). A relation of the form $y = a + b(\frac{1}{x})$ is suggested.

4 (*a*) Plot y against $\frac{1}{x}$.

4 (*b*) Find a relation approximately expressing y in terms of x.

17. Using the same axes, and the same scales on both the x and the y axes throughout, draw the graphs of

4 (*a*) $y = f(x)$ where $f(x) = (x - 1)^2(x + 2)$;

2 (*b*) $y = f'(x) = \frac{d}{dx}\left\{(x - 1)^2(x + 2)\right\}$

1 (*c*) $y = f''(x) = \frac{d}{dx}\left[\frac{d}{dx}\left\{(x - 1)^2(x + 2)\right\}\right]$

3 (*d*) What information about $f(x)$ is given by the graph of $f'(x)$?

2 (*e*) What information about $f'(x)$ is given by the graph of $f''(x)$?

HIGH SCHOOL AND UNIVERSITY MATRICULATION EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1941

ALGEBRA 2

Time—3 hours.

Note—You are not expected to answer all the questions on this paper, but answer as many as you can.

Questions may be done in any order. Except in the first question, complete solutions should be shown.

Graph paper and mathematical tables will be supplied by the presiding examiner.

Values

1. In this question, answers only are required. On your foolscap, list answers thus:

(a) ....

(b) ....

etc.

1 (a) $\frac{d}{dx}x^3 =$ ....

1 (b) $\int x^3 dx =$ ....

1 (c) t varies as r. If r increases, t ....

1 (d) The maximum value of $5 - x^2$ is ....

1 (e) $y = x^{-\frac{2}{3}}$. If $x = 8$, $y =$ ....

1 (f) If $f(x) = \log x$, then $f(1) =$ ....

1 (g) In the graph of $y = \frac{1}{x}$, the sign of the slope is ....

1 (h) As x increases without limit, $\frac{2x - 3}{5x}$ approaches the limit ....

1 (i) If $\frac{dy}{dx} = 0$ at a given point on a curve, then the tangent to the curve at that point is ....

1 (j) If $\frac{dy}{dx}$ is negative, then y is a .... function.

2. A bullet, fired vertically upwards, rises s feet in t seconds, where $s = 500\,t - 16\,t^2$.

2 (a) When $t = 4$ and $\triangle t = 0.1$, evaluate $\frac{\triangle s}{\triangle t}$.

1 (b) When $t = 4$, evaluate $\frac{ds}{dt}$

3 (c) In terms of the motion of the bullet, interpret $\frac{\triangle s}{\triangle t}$ and $\frac{ds}{dt}$, attaching the correct unit in each instance.

Values

2 **3.** If a, 3, 4, b are in continued proportion, find $\dfrac{a}{b}$.

4. Expressing results without negative or fractional exponents, find $\dfrac{dy}{dx}$ when

3 (*a*) $y = \dfrac{x^3}{6} + 4\sqrt{x} - \dfrac{1}{2x^2}$.

3 (*b*) $y = 2\sqrt{x^3} + \dfrac{3}{\sqrt[3]{x}}$.

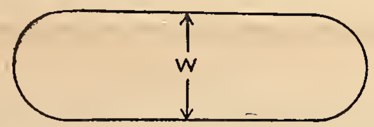
5. A varies as $\dfrac{W}{v^2}$ where A sq. ft. is the area of the wing surface of an aeroplane, W tons is its total weight (aeroplane and load), and v miles per hour is its speed.

3 (*a*) Find the ratio of the wing areas of two aeroplanes, each having a total weight of 3 tons, the first having a speed of 100 miles per hr. and the second a speed of 120 miles per hr.

3 (*b*) One aeroplane has a wing area of 810 sq. ft. and a speed of 150 miles per hr. A second aeroplane has a wing area of 750 sq. ft. and a speed of 180 miles per hr. Find the ratio of the total weight of these aeroplanes.

3 (*c*) If the total weight of an aeroplane is increased in the ratio 16 : 9, how must the speed be altered, and in what ratio, so that the wing area need not be changed?

6. A design for a flower bed is shown in the following figure, which consists of a rectangle and two semi-circles. Its perimeter is 100 ft.



4 (*a*) If the area of the figure is A sq. ft. and its width W ft., express A as a function of W.

2 (*b*) If $W = 6$, compute the area of the bed to the nearest square foot.

2 (*c*) Show the range of values of W thus ....< W <....................

1 **7.** (*a*) Under what conditions is $1 + nx$ a close approximation for $(1 + x)^n$?

(*b*) Use (*a*) to find approximate values of

1 (1) $\dfrac{1}{\sqrt{0.98}}$

3 (2) $\sqrt[3]{27.81}$

3 **8.** (*a*) If $\dfrac{dy}{dx} = 2x^3 + \dfrac{2}{x^3}$, and $y = 7$ when $x = 2$, express y as a function of x.

3 (*b*) Solve: $\dfrac{ds}{dt} = \sqrt{t^3} + \dfrac{1}{\sqrt[3]{t}}$

9. If $y = 1 + 3x - 5x^2$, and x changes from 1 to 1.02,

2 (*a*) Find the exact value of Δy.

2 (*b*) Use differentiation to find an approximate value for Δy.

1 (*c*) Calculate, correct to one decimal place, the percentage error in the approximation obtained in (*b*).

10. Four boys and five girls are eligible to hold office in a student organization.

3 (*a*) In how many ways can an executive consisting of president, vice-president and secretary be chosen?

Values

3 (*b*) In how many ways can this executive be chosen if the president must be a boy and the vice-president a girl?

8 (*c*) In how many ways can a social committee of five members be chosen if there must be at least one boy on the committee?

3 **11.** (*a*) Evaluate $\displaystyle\int_{-3}^{1} (x^2 + 1)\,dx$.

2 (*b*) Interpret geometrically your result in (*a*), using a graph sketch.

12. In the expansion of $\left(2x - \dfrac{y}{2}\right)^7$,

1 (*a*) How many terms contain both x and y?

1 (*b*) What is the exponent of y in the term containing x^3?

3 (*c*) Write down the 4th term in the form $K\,x^m\,y^n$.

13. $y = 6x - x^2 - 5$

1 (*a*) Express this function in its factored form.

1 (*b*) Sketch its graph.

2 (*c*) Determine the gradients of the tangents to the curve where it crosses the x-axis.

2 (*d*) Calculate the area of the triangle formed by the tangents mentioned in (*c*) and the x-axis.

3 **14.** A water pitcher is 6 inches high and holds one pint. Another pitcher of the same shape is 9 inches high; find its capacity.

4 **15.** (*a*) The 8th and 13th terms of an A.P. are 25 and 40 respectively. Find the 10th term.

5 (*b*) The sum of the first n terms of a series is $8 - \dfrac{1}{2^{n-3}}$

Write down the first three terms.

16. The following table shows a few of the results obtained in an experiment:

x	2.5	3.2	3.8	4.5	5.0
y	0.5	4.0	11.3	14.2	19.5

3 (*a*) On graph paper, plot y against x^2.

1 (*b*) Assuming that there is a linear relationship between y and x^2, state which pair of results is most in error.

2 (*c*) Excluding the point indicated in (*b*), draw the best fitting straight line among the remaining points.

2 (*d*) From this line, work out an equation in the form $y = ax^2 + b$.

1 (*e*) When $x = 4$, what is the value of y?

1 (*f*) When $x = 4$, what is the value of $\dfrac{dy}{dx}$?

17. $y = 4x + 2 + \dfrac{1}{x}$

4 (*a*) For the following ranges, state whether $\dfrac{dy}{dx}$ is positive or negative:

(1) $-3 < x < -\frac{1}{2}$

(2) $-\frac{1}{2} < x < -\frac{1}{4}$

(3) $\frac{1}{4} < x < \frac{1}{2}$

(4) $\frac{1}{2} < x < 3$

Values

2 (*b*) Using the results obtained in (*a*), find all the values of x at which y is maximum or minimum.

2 (*c*) Find all the maximum and minimum values of y.

2 (*d*) Sketch the graph of y against x.

3 **18.** (*a*) Find the cash value of $500.00 due in 10 years if money is worth 3%. Give a complete logarithmic solution.

6 (*b*) Equal sums of money invested annually for 7 years amount to $3,206.00 one year after the last investment. If interest is calculated at 5% compounded annually, find the annual investment. Give a complete logarithmic solution.

2 **19.** (*a*) Name the curve whose equation is $y = \pm\sqrt{9 - x^2}$.

3 (*b*) Evaluate, to four significant figures, $2\int_0^3 \pi(9 - x^2)\,dx$.

2 (*c*) What is the geometrical significance of the result obtained in (*b*)?

6 **20.** (*a*) Sketch the graphs of the functions

(1) $x^2 - 5x + 6$

(2) $x^2 - 6x + 9$

(3) $x^2 + x + 2$

3 (*b*) Consider the equations

(4) $x^2 - 5x + 6 = 0$

(5) $x^2 - 6x + 9 = 0$

(6) $x^2 + x + 2 = 0$

What do the graphs (1), (2) and (3) tell you about the roots of the equations (4), (5) and (6)?

HIGH SCHOOL AND UNIVERSITY MATRICULATION EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1942

ALGEBRA 2

Time—3 hours.

Note—All answers are to be written in this booklet.

Ample space has been provided for rough work. Page 28 may be used for rough work for the exercises of page 1.

It is not expected that you will have sufficient time to do all the questions, but answer as many as you can.

In questions 1 to 25, answers only are required. Place these in the brackets. In questions 26 to 56, complete solutions should be shown in the spaces provided.

Mathematical tables will be supplied by the presiding examiner.

Values

2 **1.** If $5y = 4x$, find the value of the ratio $y : x$. (..........)

2 **2.** $a : b = c : d$

Write a fraction equal to $\frac{a}{b}$. (..........)

Write a fraction equal to $\frac{a}{c}$. (..........)

2 **3.** L varies inversely as W. How is L affected when W is halved? (..........)

2 **4.** What value of x will make the function $\frac{x + 3}{5 - x}$ equal to zero? (..........)

2 **5.** For what value of x is the function in question 4 not defined? (..........)

2 **6.** Express the perimeter (p inches) of a semicircle as a function of its radius (r inches). (..........)

2 **7.** If $f(x) = 5x - 2x^3$, find the value of $f(-2)$. (..........)

2 **8.** As h approaches the limit 0, what limit does $\frac{5h + h^2}{h}$ approach? (..........)

2 **9.** What is the gradient of the graph of the function $3x + 5$? (..........)

2 **10.** Given $y = 5x^2 + 7$. Find $\frac{d^2y}{dx^2}$. (..........)

2 **11.** At what value of x does x^2 change from a decreasing to an increasing function? (..........)

FOR ROUGH WORK.

Values

2 **12.** Find a value of x at which the slope of the graph of $x^2 - 4x$ is equal to zero. (..........)

2 **13.** $\frac{dy}{dx} \triangle x$ is often used as an approximation for $\triangle y$. For which of the following values of $\triangle x$ would you expect the approximation to be closest: 0.015, 0.021, 0.02 ? (..........)

2 **14.** Solve $\frac{ds}{dt} = t^2$. (..........)

2 **15.** Solve $\frac{ds}{dt} = \frac{1}{t^2}$. (..........)

2 **16.** Evaluate $\int_1^3 2x\,dx$. (..........)

2 **17.** Write the fifth term in the following sequence:

$\frac{1}{2}, \frac{4}{5}, \frac{9}{10}, \frac{16}{17},$ .... (..........)

2 **18.** The nth term of an *A. P.* is $\frac{6n + 19}{4}$. Write the 10th term. (..........)

2 **19.** The first two terms of a *G. P.* are 15 and 10. Write the third term. (..........)

2 **20.** Given $f(x) = (1.03)^x$. Use tables to find the value of $f(21)$ correct to the second decimal place. (..........)

2 **21.** How many different committees consisting of one man and one woman can be chosen from a group of 5 men and 6 women? (..........)

2 **22.** Evaluate $\frac{10!}{6!\ 4!}$ (..........)

2 **23.** If $(x + 5)(2x + 3)(3x + 2)(5x + 1)$ is expanded and arranged in descending powers of x, what are the first and the last terms?

(first) (..........)

(last) (..........)

2 **24.** Using a table of logarithms, solve for x: $10^x = 37.5$. (..........)

2 **25.** What is the gradient of the line joining the points (2,1) and (5,6) ? (..........)

FOR ROUGH WORK.

Values

In questions 26 to 56, complete solutions should be shown in the spaces provided.

5 **26.** The ratio of the volumes of two circular cylinders is 15 : 14 and the ratio of their heights is 5 : 21. Find the ratio of their diameters.

4 **27.** The time of oscillation of a pendulum varies as the square root of its length. A pendulum 24 inches long makes one complete oscillation in $1\frac{1}{2}$ seconds. Calculate to the nearest hundredth of an inch the length of a pendulum which makes one complete oscillation in one second.

FOR ROUGH WORK.

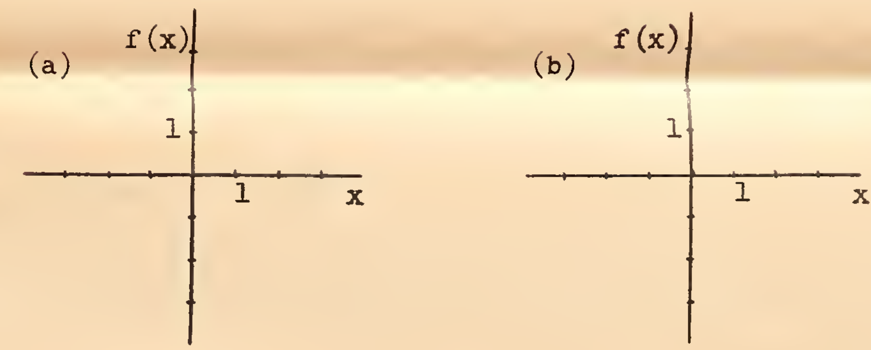
Values

4 **28.** When $s = 2$, $V = \dfrac{1}{\sqrt{t}}$ and when $t = 4$, $V = \dfrac{s^2}{8}$

Derive a formula expressing V in terms of s and t.

5 **29.** Sketch the graph of (*a*) $\dfrac{x+1}{x-1}$ and (*b*) $\dfrac{x+1}{1-x}$.

Use the sets of axes provided below.



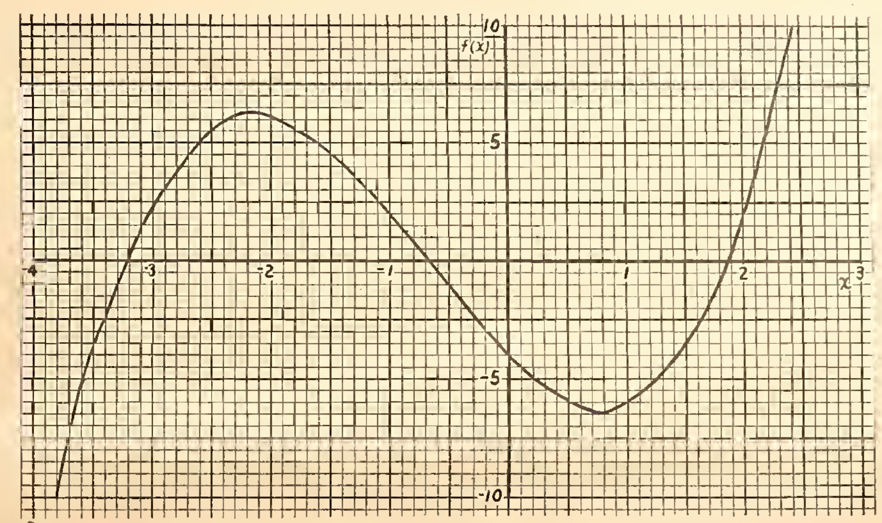
6 **30.** A right triangle has a perimeter of x inches, and one of the sides about the right angle is 4 inches long. Express the area of the triangle (A sq. in.) as a function of x.

FOR ROUGH WORK.

Values

2 **31.** The diagram below shows the graph of $x^3 + 2x^2 - 5x - 4$.

Use this graph to solve the equation $x^3 + 2x^2 - 5x - 4 = 0$.
Give answers to the nearest tenth.



3 **32.** From the graph given in question 31, solve the equation $x^3 + 2x^2 - 5x + 1 = 0$. Give answers to the nearest tenth.

FOR ROUGH WORK.

Values

3 **33.** On the set of axes used for the graph in question 31, draw the graph of $2x + 1$. Show values used for plotting in the table provided below.

x	
$2x + 1$	

3 **34.** Using the original graph given in question 31 and the graph which you were directed to draw in question 33, solve the equation $x^3 + 2x^2 - 7x - 5 = 0$. Give answers to the nearest tenth.

2 **35.** Consider functions of the form $ax^2 + bx + c$. How are the graphs of such functions restricted in shape or position when $a = 0$, $b \neq 0$, $c \neq 0$?

FOR ROUGH WORK.

Values

3 **36.** As in question 35, how are the graphs restricted when $a \neq 0$, $b = 0$, $c \neq 0$?

2 **37.** As in question 35, how are the graphs restricted when $a \neq 0$, $b \neq 0$, $c = 0$?

FOR ROUGH WORK.

Values

3 **38.** For what range of values of K are the roots of the equation $2x^2 - 3x + K = 0$ real and unequal?

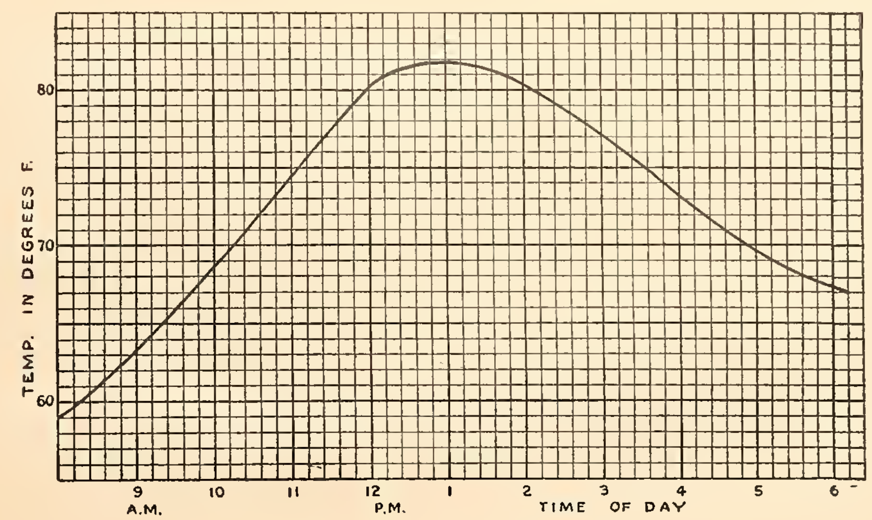
3 **39.** The function $2x^2 + x - 1$ may be expressed in the form $2(x + A)(x + B)$. Find A and B.

3 **40.** As n increases without limit, the functions $\frac{n^2}{n^2 + 1}$ and $\frac{n^2}{n^2 - 1}$ both approach the limit $+1$. How do they differ in their manner of approaching this limit?

FOR ROUGH WORK.

Values

2 **41.** The accompanying diagram shows the variations in temperature at a certain place during part of a summer day. Find the average rate of change of the temperature between 11 a.m. and 2 p.m.



3 **42.** From the diagram given in question 41, estimate the rate of change of temperature at 1:30 p.m.

4 **43.** If $y^2 = x^3$, find the value of $\frac{dy}{dx}$ when $x = 4$.

FOR ROUGH WORK.

Values

6 **44.** Find all the maximum and minimum values of $4x^3 + 15x^2 - 18x + 7$.

3 **45.** The volume of a sphere is $\frac{4}{3}\pi r^3$. If the radius of the sphere is increased from 5 cm. to 5.03 cm. use the theorem mentioned in question 13 to obtain an approximate value for the increase in the volume of the sphere. Give the answer correct to two decimal places.

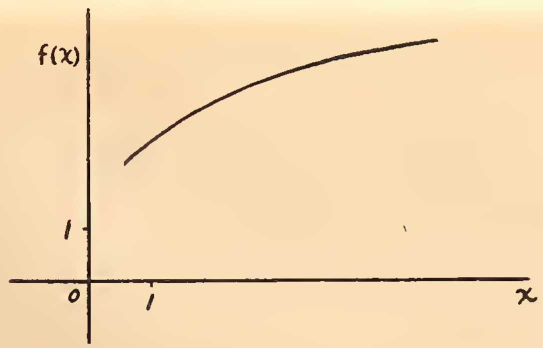
FOR ROUGH WORK.

Values

4 **46.** Given $\frac{d^2s}{dt^2} = 3t + \frac{1}{2}$. Also, when $t = 2$, $s = 6$ and $\frac{ds}{dt} = 10$.

Express s as a function of t.

2 **47.** Using the sketch below, mark the boundary of the area $\int_a^b f(x)\,dx$ with four heavy lines.



6 **48.** A sphere having a diameter of 10 inches is divided into two parts by a plane which is 3 inches from the centre of the sphere. Use integration to find the volume (correct to one decimal) of the smaller part.

FOR ROUGH WORK.

Values

2 **49.** The first term of an *A. P.* is 3 and the seventeenth term is 27. Find the sum of these seventeen terms.

5 **50.** If the sum of the first n terms of the *A. P.* in question 49 is 486, find n.

4 **51.** Find (to the nearest cent) the interest earned by investing $250.00 for 7 years at 3 % per annum compounded semi-annually.

FOR ROUGH WORK.

Values

6 **52.** A \$50.00 bond bearing interest at 3% is redeemable at par in 10 years. Find to the nearest cent the cash value of this bond if the current discount rate is $2\frac{1}{2}\%$.

5 **53.** There are 17 different books from which the winner of a contest may select the 3 to which he is entitled. Of these 17, 10 are fiction and 7 are non-fiction. If at least one of the books chosen must be non-fiction, in how many ways can the 3 books be selected?

FOR ROUGH WORK.

4 **54.** Write down and simplify the first four terms in the expansion of $\left(x - \frac{y}{2}\right)^{16}$.

3 **55.** Write the middle term in the expansion of the binomial named in question 54.

When $x = 2$ and $y = 1$, what is the value of this term?

3 **56.** Assuming that, under certain conditions,
$(1 + x)^n \simeq 1 + nx$, find an approximate value for $\frac{1}{\sqrt{104}}$.

FOR ROUGH WORK.